STAT 1222 SPRING 2006
Common Final Exam May 4, 2006

Please print the following information:

Name: ___________________________   Instructor: ___________________________

Student ID #: ______________________   Section/Time: __________________________

THIS EXAM HAS TWO PARTS

PART I. Consists of 30 multiple choice questions worth a total of 60 points. Read all questions carefully. You may do calculations on the test paper. Mark the number of the opscan sheet corresponding to the test question number with a Number 2 pencil or a mechanical pencil with HB lead. Mark only one answer; otherwise the answer will be counted as incorrect. In case there is more than one answer, mark the best answer. Please make sure that your name appears on the opscan sheet in the spaces provided.

PART II. This part consists of 3 questions (40 points in total). You MUST show all work for each question in the space provided to receive full credit for that question. If you write your explanations in another part of the test, please indicate accordingly.

At the end of the examination, you MUST hand in this test booklet, your answer sheet and all scratch paper.

FOR DEPARTMENTAL USE ONLY:
PART II:

<table>
<thead>
<tr>
<th>Questions</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum</td>
<td>14</td>
<td>14</td>
<td>12</td>
</tr>
<tr>
<td>Score</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Part I</th>
<th>Part II</th>
<th>Total</th>
</tr>
</thead>
</table>
The following is used for questions 1, 2 and 3.

A sample of eight resistors of a certain type resulted in the following sample resistances (ohms):

\[ 40, 43, 39, 35, 37, 43, 46, 37 \]

1. Find the median of the sample.
   (a) 36
   (b) 37.5
   (c) 39.5
   (d) 38.5
   (e) 43

2. Find the standard deviation of the sample.
   (a) 14
   (b) 98
   (c) 12.25
   (d) 3.742
   (e) 3.5

3. Find the first quartile of the sample.
   (a) 35
   (b) 39
   (c) 43
   (d) 37
   (e) 40
Use the following to answer questions 4, 5 and 6.

In a survey of 250 juniors majoring in psychology or communications at a large university, the students were asked whether or not they are happy with their majors. The following table gives the result of the survey. Assume that none of these students major in both areas.

<table>
<thead>
<tr>
<th></th>
<th>Happy</th>
<th>Unhappy</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Psychology</td>
<td>80</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Communications</td>
<td>115</td>
<td>35</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. What is the probability that a randomly selected student from this group is happy with the choice of the major?
   (a) .16
   (b) .25
   (c) .32
   (d) .84
   (e) .78

5. What is the probability that a randomly selected student from this group is neither happy with the choice of the major nor is a psychology major?
   (a) .14
   (b) .59
   (c) .32
   (d) .41
   (e) .60

6. What is the probability that a randomly selected student from this group is unhappy with the choice of major or is a communications major?
   (a) .82
   (b) .59
   (c) .68
   (d) .84
   (e) .23
The following is used for questions 7, 8 and 9.

A study indicates that the weights of adults are normally distributed with a mean $\mu$ of 143 lbs and a standard deviation $\sigma$ of 29 lbs.

7. What is the probability that a randomly selected adult weighs between 128 and 155 lbs?
   (a) .3576
   (b) .1591
   (c) .1985
   (d) .6985
   (e) .2347

8. If 300 adults are randomly selected, how many will weigh more than 160 pounds.
   (a) 67
   (b) .7224
   (c) 300
   (d) 150
   (e) 83

9. Find a value of weight $x$ such that only 15% of adults weigh less than that.
   (a) 121.56
   (b) 112.84
   (c) 143
   (d) 173.16
   (e) 167.44
10. Which of the following statements are true about the sampling distribution of $\bar{x}$?

I. The mean of the sampling distribution is equal to the mean of the population.

II. The standard deviation of the sampling distribution is equal to the population standard deviation divided by square root of the sample size.

III. The shape of the sampling distribution is always approximately normal.

(a) I only
(b) II only
(c) III only
(d) I and II only
(e) I and III only.

The following is used for questions 11 and 12.

Scores for men on the verbal portion of the SAT-I test are normally distributed with a mean of 509 and a standard deviation of 108. A random sample of 36 men is selected.

11. Identify the mean and standard error of their sample mean score, $(\mu_{\bar{x}}, \sigma_{\bar{x}})$

(a) 509, 108
(b) 509, 18
(c) 84.833, 108
(d) 509, 3
(e) 14.139, 3

12. What is the probability that their sample mean score $\bar{x}$ is greater than 540?

(a) .9573
(b) .5427
(c) .6554
(d) .0427
(e) .17
13. Suppose you want to obtain an estimate of the mean caffeine content in a cup of coffee correct to within 3 mg and with 80% confidence. Find the minimum sample size necessary to estimate the mean caffeine content in a cup of coffee. Assume that the population is normally distributed with a standard deviation of 30.

(a) \( n = 271 \)
(b) \( n = 3855 \)
(c) \( n = 164 \)
(d) \( n = 664 \)
(e) \( n = 2401 \).

The following is used for questions 14 and 15.

A random sample of elementary school children in New York state is to be selected to estimate the proportion \( p \) who have received a medical examination during the past year. A random sample of 200 elementary school children indicated that 18 of them had received a medical examination in the past year.

14. Find point estimate for \( p \) and also construct a 95% confidence interval for \( p \).

(a) \( .09, (.04, .14) \)
(b) \( .9, (.01, .17) \)
(c) \( 200, (.01, .17) \)
(d) \( 200, (.05, .13) \)
(e) \( .09, (.05, .13) \).

15. Using the information from the above sample, find the minimum sample size needed to estimate the population proportion \( p \) with 99% confidence. The estimate must be accurate to within .02 of \( p \).

(a) \( n = 1701 \)
(b) \( n = 4145 \)
(c) \( n = 1358 \)
(d) \( n = 2936 \)
(e) \( n = 2401 \).
The following is used for question 16.
Starting salaries of 50 college graduates who have taken a statistics course has a mean of $42,786 and a standard deviation of $8,912.

16. Construct a 97% confidence interval for the mean starting salary. Also, report the critical value $z_c$ corresponding to a confidence level of $c = .97$.
   (a) (40051.05, 45520.95), 2.17
   (b) (38257.15, 44512.85), 2.17
   (c) (40315.72, 45256.28), 1.96
   (d) (40164.48, 45407.52), 2.08
   (e) (38146.26, 46128.74), 2.08

17. In the context of statistical hypothesis testing, a Type I error is:
   (a) Failing to reject the null hypothesis when it is false
   (b) Failing to reject the alternative hypothesis when it is false
   (c) Rejecting the null hypothesis when it is true
   (d) Failing to reject the alternative hypothesis when it is true
   (e) All of the above
The following is used for questions 18, 19 and 20.

Randomly selected $n_1 = 55$ student cars have ages with a mean $\bar{x}_1$ of 5.9 years and a standard deviation $s_1$ of 3.7 years, while randomly selected $n_2 = 60$ faculty cars have ages with a mean $\bar{x}_2$ of 7.1 years and a standard deviation $s_2$ of 3.6 years.

18. Choose the correct hypotheses to test the claim that faculty cars are older than student cars. Denote by $\mu_1$ the mean age of student cars and by $\mu_2$ the mean age of faculty cars.

(a) $H_0 : \mu_1 \geq \mu_2$ versus $H_a : \mu_1 < \mu_2$

(b) $H_0 : \bar{x}_1 \geq \bar{x}_2$ versus $H_a : \bar{x}_1 < \bar{x}_2$

(c) $H_0 : \mu_1 \leq \mu_2$ versus $H_a : \mu_1 > \mu_2$

(d) $H_0 : \mu_1 = \mu_2$ versus $H_a : \mu_1 \neq \mu_2$.

(e) $H_0 : \mu_1 < \mu_2$ versus $H_a : \mu_1 \geq \mu_2$.

19. Find the value of the standardized test statistic.

(a) .5134

(b) -1.354

(c) -1.257

(d) -1.760

(e) 4.285

20. Find the rejection region and state your conclusion at $\alpha = .01$.

(a) Rejection Region: $z < -2.779$; Decision: Fail to Reject $H_0$.

(b) Rejection Region: $z < -2.33$; Decision: Fail to reject $H_0$.

(c) Rejection Region: $z > 2.779$; Decision: Reject $H_0$.

(d) Rejection Region: $z < 2.33$; Decision: Reject $H_0$.

(e) Rejection Region: $z > -2.779$; Decision: Reject $H_0$. 
The following is used for questions 21, 22 and 23.

Six randomly selected people took an IQ test A, and the next day they took a very similar IQ test B. Their scores are shown in the table below.

<table>
<thead>
<tr>
<th>Student</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test A</td>
<td>121</td>
<td>93</td>
<td>71</td>
<td>119</td>
<td>104</td>
<td>100</td>
</tr>
<tr>
<td>Test B</td>
<td>121</td>
<td>91</td>
<td>72</td>
<td>122</td>
<td>108</td>
<td>100</td>
</tr>
</tbody>
</table>

Assume that the scores in Test A and Test B are normally distributed.
Let \( d = \) Test A score \( - \) Test B score and denote by \( \mu_d \) the mean of the differences.

21. On average, do people score better on the second test than in the first test they take?
Choose the appropriate hypotheses to test the claim.
   (a) \( H_0 : \mu_d \geq 0 \) versus \( H_a : \mu_d < 0 \).
   (b) \( H_0 : \bar{d} \leq 0 \) versus \( H_a : \bar{d} > 0 \).
   (c) \( H_0 : \mu_d \leq 0 \) versus \( H_a : \mu_d > 0 \).
   (d) \( H_0 : \mu_d = 0 \) versus \( H_a : \mu_d \neq 0 \).
   (e) \( H_0 : \mu_d < 0 \) versus \( H_a : \mu_d \geq 0 \).

22. Calculate the value of the standardized test statistic.
   (a) -1
   (b) 1.237
   (c) -1.118
   (d) 1.118
   (e) -1.237

23. Find the rejection region and state your decision at \( \alpha = .05 \).
   (a) Rejection Region: \( t < -2.015 \); Decision: Fail to reject \( H_0 \).
   (b) Rejection Region: \( t < 2.015 \); Decision: Reject \( H_0 \).
   (c) Rejection Region: \( t < -1.943 \); Decision: Fail to reject \( H_0 \).
   (d) Rejection Region: \( t < 1.943 \); Decision: Reject \( H_0 \).
   (e) Rejection Region: \( t < -2.132 \); Decision: Fail to reject \( H_0 \).
The following is used for questions 24, 25 and 26.
Let \( x \) denote the number of potential weapons detected by a metal detector at an airport on a given day. The probability distribution of \( x \) is

\[
\begin{array}{c|c}
 x & P(x) \\
\hline
 0 & .14 \\
 1 & .28 \\
 2 & .22 \\
 3 & .18 \\
 4 & .12 \\
 5 & .06 \\
\end{array}
\]

24. What is the probability that at least 3 potential weapons are detected?
   (a) .18
   (b) .30
   (c) .12
   (d) .36
   (e) .06.

25. Find the mean number of potential weapons detected.
   (a) 2.04
   (b) 3
   (c) 2.5
   (d) 3.12
   (e) 2.16.

26. What is the variance of the random variable, \( x \)?
   (a) .12
   (b) 1.126
   (c) 2.038
   (d) 1.56
   (e) .752.
The following is used for questions 27, 28 and 29.
Golf-course designers have become concerned that old courses are becoming obsolete since new technology has given golfers the ability to hit the ball so far. Designers, therefore, have proposed that new golf courses need to be built expecting that the average golfer can hit the ball more than 230 yards on average. A random sample of 177 golfers show that their mean driving distance is 230.7 yards, with a standard deviation of 41.8.

27. Set up the null and alternative hypotheses to test for the designers belief.
   (a) \( H_0 : \mu \leq 230.7 \) versus \( H_a : \mu > 230.7 \)
   (b) \( H_0 : \mu = 230.7 \) versus \( H_a : \mu \neq 230.7 \)
   (c) \( H_0 : \mu \geq 230.7 \) versus \( H_a : \mu < 230.7 \)
   (d) \( H_0 : \mu \leq 230 \) versus \( H_a : \mu > 230 \)
   (e) \( H_0 : \mu \geq 230 \) versus \( H_a : \mu < 230 \)

28. Find the value of the standardized test statistic.
   (a) .7
   (b) .223
   (c) .125
   (d) -.7
   (e) -.125

29. Find the P-value for the above mentioned test.
   (a) .0871
   (b) .5871
   (c) .0228
   (d) .9772
   (e) .4129
The following is used for question 30.
The equation of the best fit line relating the amount of bill (in dollars) $x$ to the amount of tip (in dollars) $y$ is

$$\hat{y} = 0.18x - 2.70$$

30. Predict the amount of tip if the amount of bill is $70.
   (a) The predicted amount of tip is $12.60
   (b) The predicted amount of tip is $10.50
   (c) The predicted amount of tip is $9.90
   (d) The predicted amount of tip is $14

End of Multiple Choice Section
1. The table below reports the ages (in years) and the number of hours of sleep in one night by seven adults.

<table>
<thead>
<tr>
<th>Age, $x$</th>
<th>Hours of sleep, $y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>7</td>
</tr>
<tr>
<td>20</td>
<td>9</td>
</tr>
<tr>
<td>59</td>
<td>5</td>
</tr>
<tr>
<td>42</td>
<td>6</td>
</tr>
<tr>
<td>68</td>
<td>5</td>
</tr>
<tr>
<td>38</td>
<td>8</td>
</tr>
<tr>
<td>75</td>
<td>4</td>
</tr>
</tbody>
</table>

$n = 7, \sum x = 337, \sum x^2 = 18563, \sum y = 44, \sum y^2 = 296, \sum xy = 1916.$

(a). (2 pts.) Calculate the sample correlation coefficient $r$ between age, $x$, and the number of hours slept in one night, $y$.

(b). (5 pts.) Test for significant linear correlation between age of an adult and the number of hours slept in one night by an adult at $\alpha = .05$
(c). (5 pts.) Find the equation of the best fit line relating the number of hours slept by an adult, y to the adult’s age, x.

(d) (2 pts.) Can you use the equation in part (c) to predict y when x = 10? Why or why not?

2. A company claims that its medicine (Brand A) provides faster relief from pain than its competitor (Brand B). To test this claim, Brand A was given to a random sample of \( n_1 = 20 \) patients and Brand B was given to another random sample of \( n_2 = 10 \) patients. The mean times to relief for each group was found to be \( \bar{x}_1 = 44 \) and \( \bar{x}_2 = 49 \) minutes and the standard deviations were found to be \( s_1 = 11 \) and \( s_2 = 9 \), respectively. Assume that both the populations of times to relief are normally distributed with equal population standard deviations.

(a) (3 pts.) State the null and the alternate hypotheses to test the company’s claim.
(b) (5 pts.) Find the standardized test statistic.

(c) (3 pts.) State the rejection region at \( \alpha = .01 \)

(d) (3 pts.) At \( \alpha = .01 \), does the data support the company’s claim? Why or why not?
3. At a trade show, a random sample of $n = 50$ attendees were interviewed. Out of the 50 attendees who were interviewed, 30 said they were more likely to visit an exhibit when there is a giveaway. At $\alpha = .03$, test the claim that more than 52% of the attendees at trade shows were more likely to visit an exhibit when there is a giveaway.

(a)  (2 pts.) State the null and the alternative hypotheses.

$H_0 :$  

$H_a :$  

(b)  (5 pts.) Find the value of the standardized test statistic based on the observed sample.

(c)  (5 pts.) Find the critical value(s) at level $\alpha = .03$ and state the rejection rule. What is your decision?