This exam is divided into three parts. **Calculators are not allowed on Part I.** You have three hours for the entire test, but you have only one hour to finish Part I. You may start working on the other two parts of the exam whenever you are done with Part I, but you cannot use your calculator until ALL of the Part I answer sheets are collected. After these answer sheets are collected, your instructor will announce that calculators are allowed on Parts II and III.

These pages contain Part I which consists of 12 multiple choice questions. These questions must be answered without the use of a calculator.

• You must use a pencil with a soft black lead (#2 or HB) to enter your answers on the answer sheet

• For each question choose the response which best fits the question

• If you wish to change an answer, make sure that you completely erase your old answer and any other extraneous marks.

• There is no penalty for guessing

• If you mark more than one answer to a question, the question will be scored as incorrect.

• You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the answer sheet.

• Make sure that your name appears on the answer sheet and that you fill in the circles corresponding to your name.

After 1 hour, you MUST hand in the answer sheet for Part I. At the end of the exam, you MUST hand in all remaining test materials including test booklets, the answer sheet for Part II, and scratch paper.
1. Evaluate the integral \( \int (x+2)(x-3) \, dx \)
   (a) \( \left( \frac{x^2}{2} + 2x \right) \left( \frac{x^2}{2} - 3x \right) + C \)
   (b) \( 2x - 1 + C \)
   (c) \( x^3 - \frac{x^2}{2} + C \)
   (d) \( \frac{1}{3}x^3 - \frac{1}{3}x^2 - 6x + C \)
   (e) \( (x+2) \left( \frac{1}{2}x^2 - 3x \right) + (\frac{1}{2}x^2 + 2x)(x-3) + C \)

2. Let \( f(x) = \begin{cases} 2x, & x \leq 1 \\ 3x^2 - 1, & x > 1 \end{cases} \)
   Evaluate \( \int_{0}^{1} f(x) \, dx \).
   (a) 58
   (b) 59
   (c) 60
   (d) 61
   (e) 62

3. Use integration by parts to evaluate the integral \( \int x \cos(2x) \, dx \).
   (a) \( \frac{1}{2}x^2 \cos(x^2) + C \)
   (b) \( \frac{1}{4}x^2 \sin(2x) + C \)
   (c) \( \frac{1}{2} \sin(2x) - \frac{1}{2}x \cos(2x) + C \)
   (d) \( \frac{1}{2} \cos(2x) - \frac{3}{4}x \sin(2x) + C \)
   (e) \( \frac{1}{2} \cos(2x) + \frac{1}{2}x \sin(2x) + C \)

4. Let \( f(x) = \int_{1}^{x^2} \tan(t^3) \, dt \). Evaluate \( f'(x) \).
   (a) \( 2x \sec^2(x^3) \)
   (b) \( 2x \sec^2(x^6) \)
   (c) \( 2x \sec^2(x^6) \)
   (d) \( 2x \tan(x^6) - \tan(1) \)
   (e) \( 2x \tan(x^6) \)
5. Evaluate $\int \frac{x^3}{x^3 + 1} \, dx$
(a) $\frac{x^4}{x^3 + 1} + C$
(b) $\frac{1}{4} x^4 \tan^{-1}(x) + C$
(c) $\frac{1}{2} x^2 - \tan^{-1}(x) + C$
(d) $\frac{1}{2} x^2 - \frac{1}{2} \ln(1 + x^2) + C$
(e) None of these

6. If we use partial fractions and write $\frac{2x+1}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$ then
(a) $A = -2$
(b) $A = -1$
(c) $A = 1$
(d) $A = 2$
(e) $A = 3$

7. Evaluate $\int_0^{\infty} \frac{x}{(x^4 + 1)^2} \, dx$.
(a) The integral does not converge
(b) $\frac{1}{6}$
(c) $\frac{1}{3}$
(d) $\frac{1}{3}$
(e) $\frac{1}{2}$

8. Consider the integral $\int_{-1}^{2} \frac{1}{x^2} \, dx$. Which of the following statements is correct?
(a) $\int_{-1}^{2} \frac{1}{x^2} \, dx = -\frac{3}{2}$
(b) $\int_{-1}^{2} \frac{1}{x^2} \, dx = \frac{3}{2}$
(c) $\int_{-1}^{2} \frac{1}{x^2} \, dx = \ln(4)$
(d) $\int_{-1}^{2} \frac{1}{x^2} \, dx = -\frac{1}{2}$
(e) $\int_{-1}^{2} \frac{1}{x^2} \, dx$ is a divergent improper integral.
9. We wish to evaluate the integral \( \int \sqrt{5 + 4x - x^2} \, dx \) by completing the square and then using one of the formulas below.

\[
I. \int \sqrt{a^2 - u^2} \, du = \frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \left( \frac{u}{a} \right) + C
\]

\[
II. \int \sqrt{u^2 - a^2} \, du = \frac{u}{2} \sqrt{u^2 - a^2} - \frac{a^2}{2} \ln |u + \sqrt{u^2 - a^2}| + C
\]

\[
III. \int \sqrt{u^2 + a^2} \, du = \frac{u}{2} \sqrt{u^2 + a^2} + \frac{a^2}{2} \ln \left( u + \sqrt{u^2 + a^2} \right) + C,
\]

Which of the following statements is correct?
(a) Use \( I \) with \( u = x - 2 \) and \( a = 3 \)
(b) Use \( I \) with \( u = x - 2 \) and \( a = \sqrt{5} \)
(c) Use \( II \) with \( u = 2 - x \) and \( a = 1 \)
(d) Use \( II \) with \( u = x - 2 \) and \( a = 3 \)
(e) Use \( III \) with \( u = x + 2 \) and \( a = 9 \)

10. A particle is moved along the \( x \)-axis by a force that measures \( 50e^{-2x} \) pounds at a point \( x \) feet from the origin. Find the work done in moving the particle from \( x = 0 \) to \( x = 4 \) feet.

(a) \( 25 - 25e^{-8} \) ft-lbs
(b) \( 50e^{-8} - 50 \) ft-lbs
(c) \( 50 + \frac{50}{7}e^{-3} \) ft-lbs
(d) \( 25 + 25e^{-8} \) ft-lbs
(e) \( 50e^{-4} - 50 \) ft-lbs

11. What is the exact value of the sum \( \sum_{n=2}^{\infty} \frac{2^n}{3^n + 1} \)?

(a) \( \frac{2}{9} \)
(b) \( \frac{2}{3} \)
(c) \( \frac{4}{3} \)
(d) \( \frac{4}{27} \)
(e) \( \frac{8}{27} \)

12: The Taylor expansion of \( f(x) = x^3 + 3x - 2 \) about \( a = 1 \) is

(a) \( 2 + 6(x - 1) + 6(x - 1)^2 + 6(x - 1)^3 \)
(b) \( 2 + 6(x - 1) + 3(x - 1)^2 + (x - 1)^3 \)
(c) \( -2 + 3(x - 1) + 3(x - 1)^2 + (x - 1)^3 \)
(d) \( -2 + 6(x - 1) + 6(x - 1)^2 + 6(x - 1)^3 \)
(e) \( 2 + 3(x - 1) \)
These pages contain Part II which consists of 13 multiple choice questions. After the answer sheets for Part I have all been collected, and your instructor announces that calculators are OK, you are allowed to use a calculator on this part of the exam.

- You must use a pencil with a soft black lead (#2 or HB) to enter your answers on the answer sheet.

- For each question, choose the response which best fits the question.

- If you wish to change an answer, make sure that you completely erase your old answer and any other extraneous marks.

- There is no penalty for guessing.

- If you mark more than one answer to a question, the question will be scored as incorrect.

- You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the answer sheet.

- Make sure that your name appears on the answer sheet for Part II and that you fill in the circles corresponding to your name.

At the end of the exam, you MUST hand in all remaining test materials including test booklets, the answer sheet for Part II, and scratch paper.
1. The graph of $y = f(x)$ is shown below. Estimate the value of the integral $\int_1^5 f(x) \, dx$. (Notice the limits of integration.)

$$\begin{array}{c}
\text{(a) } 3 \\
\text{(b) } 3.5 \\
\text{(c) } 4 \\
\text{(d) } 4.5 \\
\text{(e) } 5
\end{array}$$

2. Suppose that $\int_0^2 f(x) \, dx = 3$, $\int_0^3 f(x) \, dx = 4$, and $\int_1^4 f(x) \, dx = -1$. Evaluate $\int_2^4 f(x) \, dx$.

(a) $-1$
(b) 0
(c) 1
(d) 2
(e) 3

3. Which of the following integrals is represented by the right Riemann sum

$$\left( \frac{2}{n} \right) \left( \sqrt{1 + (2/n)^2} + \sqrt{1 + (4/n)^2} + \ldots + \sqrt{1 + 2^2} \right)$$

(a) $\int_0^1 \sqrt{1 + 2x^2} \, dx$
(b) $\int_0^1 \sqrt{1 + x^2} \, dx$
(c) $\int_0^2 \sqrt{1 + x^2} \, dx$
(d) $\int_0^2 \sqrt{1 + (2x)^2} \, dx$
(e) $\int_1^3 \sqrt{1 + (2/x)^2} \, dx$
4. When we use the substitution $u = 2x + 1$, the integral $\int_0^1 (2x + 1)^7 dx$ becomes

(a) $\int_1^3 u^7 du$
(b) $\frac{1}{2} \int_1^3 u^7 du$
(c) $2 \int_1^3 u^7 du$
(d) $\int_0^1 u^7 du$
(e) $\frac{1}{2} \int_0^1 u^7 du$
5. Evaluate the area bounded by the curves $y = x^2$ and $y = 3x - 2$.

(a) $\frac{1}{6}$
(b) $\frac{1}{6}$
(c) $\frac{1}{3}$
(d) $\frac{1}{3}$
(e) $\frac{1}{2}$

6. Which of the following sequences converge?

I. $\left\{ \frac{n}{2^{n+1}} \right\}$  
II. $\left\{ \frac{1}{\sqrt{n}} \right\}$  
III. $\left\{ \frac{(-1)^n n}{2^{n+1}} \right\}$

(a) I only  
(b) II only  
(c) III only  
(d) I and II only  
(e) I, II, and III

7. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n}{2^{n+1}}$  
II. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$  
III. $\sum_{n=1}^{\infty} \frac{(-1)^n n}{2^{n+1}}$

(a) I only  
(b) II only  
(c) III only  
(d) I and II only  
(e) I, II, and III

8. The series $\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$

(a) converges by the limit comparison test  
(b) diverges by the limit comparison test  
(c) converges by the ratio test  
(d) diverges by the ratio test  
(e) converges by the integral test

9. The series $\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$

(a) converges absolutely  
(b) converges, but not absolutely  
(c) diverges by the comparison test  
(d) diverges by test for divergence  
(e) diverges by the limit comparison test
10. Use the binomial series (or an alternative) to evaluate the coefficient of $x^3$ in the power series expansion of \((1 + x)^{-2}\) centered about $a = 0$.

(a) $-4$
(b) $-2$
(c) $2$
(d) $4$
(e) $6$

11. Calculate the $x$-coordinate of the centroid for the region bounded by $y = 0, x = 0,$ and $y = 4 - x^2$.

(a) $\frac{8}{3}$
(b) $\frac{2}{3}$
(c) $\frac{11}{16}$
(d) $\frac{3}{4}$
(e) $\frac{13}{16}$

12. Which of the following is closest to the arclength of the curve $x = 1 + 3t^2, y = 4 + 2t^3, 0 \leq t \leq 1$?

(a) 3.24
(b) 3.45
(c) 3.66
(d) 3.74
(e) 3.81

13. Suppose we want to approximate the value of $\int_1^7 \frac{1}{1+t} \, dt$ using the Trapezoidal rule. The error formula for $\int_a^b f(x) \, dx$ using this rule is $|E_T| \leq \frac{K(b-a)^3}{12n^2}$ where $|f''(x)| \leq K$ for all $x$ in the interval $[a, b]$. What is the smallest $n$ listed below that assures the error is less than 0.001?

(a) 27
(b) 37
(c) 55
(d) 68
(e) 102
These pages contain Part III which consists of 5 free response questions.

Please show all your work in this test booklet. Loose paper will not be graded.

• If you are basing your answer on a graph on your calculator, sketch this graph in the answer booklet. Be sure to label your window by putting a scale on each axis.

At the end of the exam, you MUST hand in all remaining test materials including test booklets, answer sheet, and scratch paper.

<table>
<thead>
<tr>
<th>PROBLEM</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRADE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

FREE RESPONSE SCORE: ____________________
1. Set up, but do not evaluate, an integral (or integrals) that give the area of the shaded region shown below.

(b) Suppose that the region below is revolved about the $y$-axis. Set up, but do not evaluate, an integral giving the volume of the resulting solid.
(c) The region given in part (b) is revolved about the line $y = -2$. Set up, but do not evaluate, an integral giving the volume of the resulting solid.
2. A hemispherical tank with radius 10 feet is filled with water to a level 2 feet below the top of the tank. The density of water is 62.5 pounds/ft$^3$. In this problem, we wish to determine how much work it takes to pump all the water out of a spigot level with the top of the tank.

Suppose we want to lift a slab of water that is $\Delta x$ feet thick, and $x$ feet below the top of the tank. (see the diagram below)

(a) Looking at the diagram, how long is the side of the triangle that is labeled $A$?

(b) Use the pythagorean theorem to solve for $W$ as a function of $x$ and $A$. (Be sure to use the value of $A$ from part (a).)

(c) Cross sections viewed from the top of the tank are circles. What is the (approximate) volume of the slab shown in grey above? (Recall that the slab has thickness $\Delta x$.)

(d) What is the weight of the slab in part (c)?

(e) Write an integral (including limits of integration) that expresses the amount of work required to empty the tank. Do NOT evaluate this integral. Remember that the tank is filled with water to a level that is 2 feet below the top of the tank.
3. Determine the interval of convergence for the power series \( \sum_{n=0}^{\infty} \frac{n(x+1)^n}{3^n(n^n+1)} \). Be sure to check the endpoints of the interval.
4. The base of a solid is the triangular region shown. Cross sections perpendicular to the $x$-axis are squares. Find the volume of the solid. Use the techniques of calculus, not your calculator, to evaluate the integral.
5. In this problem, we wish to approximate the value of \( \int_0^{\frac{4}{3}} \frac{x}{1+x^3} \, dx \) using a power series representation.

(a) Recall that \( \frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots \) for \( |x| < 1 \). Find a power series representation for \( \frac{1}{1+x^3} \). You need only show the first four non-zero terms.

(b) Use the result of part (a) to find a power series representation for \( \frac{x}{1+x^3} \). You need only show the first four non-zero terms.

(c) Use the series in part (b) to evaluate \( \int_0^{\frac{4}{3}} \frac{x}{1+x^3} \, dx \) with an error less than 0.01.

(d) What is the smallest number of nonzero terms in the power series for \( \frac{x}{1+x^3} \) needed to estimate \( \int_0^{\frac{4}{3}} \frac{x}{1+x^3} \, dx \) with an error less than 0.0001 = \( 10^{-4} \)? Explain your answer.