PART I (Calculators not allowed)

1. If \( g(x) = x^2 + e^x \), then \( g'(x) = \)
   \( \textbf{(a)} \) \( x^2 + e^x \)
   \( \textbf{(b)} \) \( 2x + e^x \)
   \( \textbf{(c)} \) \( x^2 + xe^{x-1} \)
   \( \textbf{(d)} \) \( 2x + xe^{x-1} \)
   \( \textbf{(e)} \) \( 2 + e^x \)

2. \( \frac{d}{dx}(x \sin x) = \)
   \( \textbf{(a)} \) \( \sin x \)
   \( \textbf{(b)} \) \( \cos x \)
   \( \textbf{(c)} \) \( -\cos x \)
   \( \textbf{(d)} \) \( \sin x - x \cos x \)
   \( \textbf{(e)} \) \( \sin x + x \cos x \)

3. If \( f(t) = \frac{\ln t}{t} \), then \( f'(2) = \)
   \( \textbf{(a)} \) \( \frac{\ln 2}{2} \)
   \( \textbf{(b)} \) \( \frac{1 - \ln 2}{4} \)
   \( \textbf{(c)} \) \( \frac{1 + \ln 2}{4} \)
   \( \textbf{(d)} \) \( \frac{1}{2} \)
   \( \textbf{(e)} \) \( \frac{1}{4} \)
4. If \( f(x) = (3x^2 + 1)^4 \), then \( f'(x) = \)
   
   (a) \( (3x^2 + 1)^4 \)
   
   (b) \( 24x(3x^2 + 1)^3 \)
   
   (c) \( 4(3x^2 + 1)^3 \)
   
   (d) \( 4(6x)^3 \)
   
   (e) \( 4(6x + 1)^3 \)

5. Which of the following is correct?
   
   (a) \( \frac{d}{dx}(\ln(3x^2 + 4)) = \ln(3x^2 + 4) \)
   
   (b) \( \frac{d}{dx}(\ln(3x^2 + 4)) = \frac{1}{3x^2 + 4} \)
   
   (c) \( \frac{d}{dx}(\ln(3x^2 + 4)) = \frac{1}{6x} \)
   
   (d) \( \frac{d}{dx}(\ln(3x^2 + 4)) = \ln 6x \)
   
   (e) \( \frac{d}{dx}(\ln(3x^2 + 4)) = \frac{6x}{3x^2 + 4} \)

6. Let \( h(s) = \sin^2(2s) \). Then \( h'(s) = \)
   
   (a) \( 4\sin(2s)\cos(2s) \)
   
   (b) \( 2\sin(2s)\cos(2s) \)
   
   (c) \( \cos^2(2s) \)
   
   (d) \( 2\cos^2(2s) \)
   
   (e) \( 2\cos(s) + 2 \)
7. Let \( f(x) = x^3 - 3x^2 + 7 \). Which of the following statements is true?

(a) \( f \) is increasing on \(( -\infty, \infty)\)
(b) \( f \) is decreasing on \(( -\infty, \infty)\)
(c) \( f \) is increasing on \(( 0, \infty)\)
(d) \( f \) is decreasing on \(( 0, 2)\)
(e) \( f \) is increasing on \(( 0, 2)\)

8. Consider the graph of the function \( f \):

Which of the following is correct?

(a) \( \lim_{x \to 1} f(x) = 4 \)
(b) \( \lim_{x \to 1} f(x) = 3 \)
(c) \( \lim_{x \to 1} f(x) = 2 \)
(d) \( \lim_{x \to 1} f(x) = 1 \)
(e) \( \lim_{x \to 1} f(x) \) does not exist

9. \( \lim_{x \to \infty} \frac{x^2 + 1}{xe^x + 1} = \)

(a) \( \infty \)
(b) 1
(c) \(-1\)
(d) 0.01
(e) 0
10. Let \( g(x) = 3x^2 + 1 \). Which of the following is the equation of the tangent line to the graph of \( g \) at \( x = 1 \)?

(a) \( y - 6 = 4(x - 1) \)
(b) \( y = 4(x - 1) \)
(c) \( y = 6(x - 1) \)
(d) \( y - 4 = 6(x - 1) \)
(e) \( y - 6 = 6(x - 1) \)

11. \( \lim_{x \to 0} \frac{2x^2 - x}{10x + 1} = \)

(a) \(-1/10\)
(b) \(0\)
(c) \(\infty\)
(d) \(1/5\)
(e) \(-\infty\)

12. Let \( f(x) = \frac{1}{x^2 + x + 3} \). Then \( f \) has a local maximum at \( x = \)

(a) \(0\)
(b) \(1/2\)
(c) \(1\)
(d) \(-1/2\)
(e) \(-1\)
13. Let \( f(x) = 6x^2 + 1 \). Which of the following is the general antiderivative of \( f \)?

(a) \( 6x^3 + 1 + C \)
(b) \( 6x^3 + x + C \)
(c) \( 12x + C \)
(d) \( 2x^3 + x + C \)
(e) \( 6x^2 + 1 + C \)

14. For a certain function \( g \), it is known that \( g'(t) = 2t + \frac{1}{t} \) for \( t > 0 \) and \( g(1) = 2 \). Which of the following is correct?

(a) \( g(t) = t^2 + \ln t + 1 \)
(b) \( g(t) = t^2 + \ln t \)
(c) \( g(t) = 2t + \ln t \)
(d) \( g(t) = 2t + \ln t + 1 \)
(e) \( g(t) = t^2 + \ln t + 2 \)

15. Let \( f(x) = x^3 - 3x^2 + 7 \). Which of the following statements is true?

(a) The graph of \( f \) is concave up on \( (-\infty, \infty) \)
(b) The graph of \( f \) is concave down on \( (-\infty, \infty) \)
(c) The graph of \( f \) is concave down on \( (-\infty, 1) \) and concave up on \( (1, \infty) \)
(d) The graph of \( f \) is concave up on \( (-\infty, 1) \) and concave down on \( (1, \infty) \)
(e) The graph of \( f \) is concave down on \( (-\infty, 0) \) and concave up on \( (0, \infty) \)
16. Let \( g(x) = \frac{1}{x} \). Then \( g'(x) = \\

(a) \ln x \\
(b) \frac{1}{x} \\
(c) \frac{x - 1}{x^2} \\
(d) -\frac{1}{x^2} \\
(e) 0 \)
PART II (Calculators allowed)

1. Let \( f(x) = e^{2x} \). Recall that the inverse \( f^{-1} \) stands for the inverse of the function \( f \). Which of the following is correct?
   
   (a) \( f^{-1}(x) = \frac{1}{2} \ln x \)
   
   (b) \( f^{-1}(x) = \ln x \)
   
   (c) \( f^{-1}(x) = \ln 2x \)
   
   (d) \( f^{-1}(x) = \frac{1}{2} \ln 2x \)
   
   (e) \( f^{-1}(x) = \ln(x^\frac{1}{2}) \)

2. Let \( f(x) = \sqrt{x - 2} \). The domain of the function \( f \) is
   
   (a) \( [0, \infty) \)
   
   (b) \( [1, \infty) \)
   
   (c) \( [2, \infty) \)
   
   (d) \( (-\infty, \infty) \)
   
   (e) \( (-\infty, 2] \)

3. A ball is thrown straight up into the air with an initial velocity of 50 ft/s. The height \( h(t) \) of the ball after \( t \) seconds is given by \( h(t) = 50t - 16t^2 \). What is the average velocity for the time period beginning when \( t = 1 \) and lasting 0.1 seconds?
   
   (a) 16.4 ft/s
   
   (b) 50 ft/s
   
   (c) 18 ft/s
   
   (d) 1.64 ft/s
   
   (e) 32 ft/s
4. What is the slope of the tangent line to the curve \( y^2 + xe^y = 1 \) at the point \((1, 0)\)?

(a) \(-1\)
(b) 0
(c) 1
(d) 2
(e) 3

5. Let \( f(x) = 4x^3 + 3x^2 - 6x + 3 \). The absolute minimum value of \( f \) on the interval \([0, 1]\) is

(a) 3
(b) \(\frac{5}{4}\)
(c) 4
(d) 8
(e) \(-5\)

6. Let \( f(x) = \begin{cases} x^2 - 3 & \text{if } x \leq 1 \\ 2x - c & \text{if } x > 1 \end{cases} \). Assuming that \( f \) is continuous at \( x = 1 \), what must be the value of \( c \)?

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4
7. The radius of a sphere is increasing at the constant rate of 2 in/s. How fast is the volume increasing when the radius is 8 in? (Recall that the volume $V$ of a sphere of radius $r$ is given by $V = (4/3)\pi r^3$.)

(a) 256$\pi$ in$^3$/s
(b) 512$\pi$ in$^3$/s
(c) 2 in$^3$/s
(d) 8$\pi$ in$^3$/s
(e) $\pi$ in$^3$/s

8. Let $f(x) = \sqrt{x}$. If the linear approximation of $f$ at $a = 25$ is used to approximate $f(26)$, then the approximation obtained is

(a) 4.8
(b) 4.9
(c) 5.0
(d) 5.1
(e) 5.099

9. The graph of the equation $y = \tan x$ is shifted to the left 2 units. Which of the following is the equation of the resulting graph?

(a) $y = \tan x + 2$
(b) $y = \tan x - 2$
(c) $y = \tan(x + 2)$
(d) $y = \tan(x - 2)$
(e) $y = 2\tan x$
10. Suppose that Newton’s method is used to find the real root of \( x^5 - 3 = 0 \). If the initial guess (first approximation) is \( x_1 = 1 \), which of the following is closest to the second approximation \( x_2 \)?

(a) 1.4  
(b) 1.3  
(c) 1.2  
(d) 1.1  
(e) 1.0

11. Here is the graph of the derivative \( g' \) of a function \( g \):

Based on the graph, which of the following is necessarily true about the function \( g \)? (Note that you are given the graph of \( g' \), not \( g \).)

(a) \( g \) is increasing on \((-\infty, \infty)\)  
(b) \( g \) is decreasing on \((-\infty, \infty)\)  
(c) \( g \) is decreasing on \((-\infty, 0)\) and increasing on \((0, \infty)\)  
(d) \( g \) is decreasing on \((-\infty, 1)\) and increasing on \((1, \infty)\)  
(e) \( g \) is increasing on \((-\infty, 0)\) and decreasing on \((0, \infty)\)

12. Consider the curve with parametric equations:
\[
    x = 2 \cos t, \quad y = 3 \sin t
\]

If we eliminate the parameter, the resulting Cartesian equation of the curve is

(a) \( x^2 + y^2 = 1 \)  
(b) \( x^2 + y^2 = 36 \)  
(c) \( 4x^2 + 9y^2 = 36 \)  
(d) \( 9x^2 + 4y^2 = 36 \)  
(e) \( 2x^2 + 3y^2 = 1 \)
13. At which of the following points on the curve \( y = \ln(x^2 + 6x + 11) \) is the tangent line horizontal?

(a) \((0, \ln 11)\)
(b) \((-1, \ln 6)\)
(c) \((-2, \ln 3)\)
(d) \((-3, \ln 2)\)
(e) \((-4, \ln 3)\)

14. Let \( f(x) = e^x \). Which of the following is the range of the function \( f \)?

(a) \((-\infty, \infty)\)
(b) \((-\infty, 0)\)
(c) \((-\infty, 1)\)
(d) \((1, \infty)\)
(e) \((0, \infty)\)
1. A particle moves along the $x$-axis. Its position at time $t$ is $x(t) = t^3 - 6t^2 + 1, \; t \geq 0$, where $t$ is measured in seconds and $x$ is measured in meters.

   (a) Where is the particle at time $t = 0$?

   (b) Determine the velocity of the particle at time $t$.

   (c) For which times $t$ is the particle moving to the right?

   (d) Determine the acceleration of the particle at time $t$.

   (e) For which times $t$ is the particle speeding up?
2. For a certain function $f$ the following information is known:

(i) $f$ and its first and second derivatives are continuous on $(-\infty, \infty)$;

(ii) $f'(x) < 0$ on $(-\infty, 0)$, and $f'(x) > 0$ on $(0, \infty)$;

(iii) $f''(x) > 0$ on $(-\infty, 1)$, $f''(x) < 0$ on $(1, 3)$, and $f''(x) > 0$ on $(3, \infty)$; and

(iv) $f(0) = 0$, $f(1) = 2$, and $f(3) = 5$.

(a) Find all local maximum and minimum values of $f$.

(b) Find the points of inflection.

(c) Sketch a graph which could be the graph of $f$. 

3. Let \( f(x) = e^{x^2 - 2x} \).

(a) Find the critical number of \( f \).

(b) Find the intervals where \( f \) is increasing and the intervals where \( f \) is decreasing. Use calculus to justify your conclusions.

(c) Find any local maximum and minimum values of \( f \).
4. Consider the following optimization problem: Find the point on the curve \( y = \sqrt{x} \) which is closest to the point \((1, 0)\).

   (a) Let \( D \) denote the distance from \((1, 0)\) to a point \((x, y)\) on the curve \( y = \sqrt{x} \). Express \( D \) as a function of \( x \).

   (b) Find the critical number of the function \( D \).

   (c) Explain why \( D \) is minimized at this critical number. [Suggestion: Do a sign analysis of \( D' \).]

   (d) Which point on the curve \( y = \sqrt{x} \) is closest to \((1, 0)\)?
5. Recall that $\sin^{-1}$ denotes the arcsine function (that is, the inverse of the sine function).

(a) What is \( \frac{d}{dx}(\sin^{-1} x) \)?

(b) Use the result of part (a) to find \( \lim_{h \to 0} \frac{\sin^{-1}(0.5 + h) - \sin^{-1} 0.5}{h} \).
Key to Part I:

1. b
2. e
3. b
4. b
5. e
6. a
7. d
8. a
9. e
10. d
11. b
12. d
13. d
14. a
15. c
16. d
Key to Part II:

1. a
2. c
3. a
4. a
5. b
6. e
7. b
8. d
9. c
10. a
11. c
12. d
13. d
14. e