Exploring alternative mathematics assessments with Latino/a students

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This study explores the wealth of student thinking generated by the task-based interviews of 15 Latino/a students as they solve measurement items from NAEP. We conjecture that these interviews could be used to complement paper-and-pencil assessments, especially in the case of Latino/a students who have been known to struggle to comprehend the language in the questions. The paper discusses three major themes namely, issues of language, use of visual cues, and mathematical communication from our interviews. Further, key factors in designing and conducting task-based interviews with students and the benefits and challenges of this form of assessment are also discussed.

Traditional paper-and-pencil tests have long been the main avenue for evaluating students’ mathematical understanding. The Assessment Principle (NCTM, 2000) states “Over reliance on such assessments [paper-and-pencil] may give an incomplete and perhaps distorted picture of students’ performance… Teachers must ensure that all students have an opportunity to demonstrate clearly and completely what they know and can do” (pp. 23-24). In the case of Latino students, who form the majority of the English Language Learners (ELLs) in the country, the language used in the test items has shown to make a significant difference in their performance (Abedi & Lord, 2001). Thus it is important to explore other means of assessing ELL Latino students that can complement the results of the paper-and-pencil assessments.

In this article we illustrate how using interviews provided students opportunities to demonstrate their mathematical knowledge and offered a wealth of information that could be used to guide further instruction. These interviews were part of a research study at the Center for the Mathematics Education of Latinos/as (CEMELA\(^1\)) that is looking into equitable ways to
assess Latino students’ understanding of measurement concepts. Although we are aware that conducting one-to-one interviews with students is quite hard for classroom teachers because of their schedule constraints, the potential gains in student knowledge and personal growth are immense.

A focus on measurement

Our motivation to examine the measurement strand were based on Lubienski’s (2003) report that this strand had the largest achievement gap when comparing NAEP 2000 scores of White and Black students and White and Hispanic students. In this article we report on two of the NAEP tasks on measurement that we used in our interviews with 15 working class Latino students in grades 4 through 6 (the triangle square (TS) problem and the area comparison (AC) problem; see Figure 1) The AC problem is also discussed in Lubienski (2003) and in Strutchens, Martin, & Kenney (2003).

The interviews

The students were encouraged to think independently about the tasks and then discuss their work with the interviewer, who asked probing questions to uncover the students’ thinking. A standard first question was “Can you explain your thinking to me?” If the student provided an incorrect or unclear explanation, particular attention was paid to understanding the students’ method, with questions such as, Do you know what the word perimeter means?, What does this sentence mean “If the square and triangle above have the same perimeter”? Do you think that these two figures, the triangle and the square, have the same perimeter?, How do you find the

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1 CEMELA is a Center for Learning and Teaching supported by the National Science Foundation, grant number ESI-0424983. The views expressed here are those of the authors and do not necessarily reflect the views of NSF. The authors wish to thank ----------------, who reviewed the manuscript and offered support and feedback.
perimeter of the square? What do you know about a square? Or more pointedly, What do you know about the sides of a square?

These questions served to highlight the students’ thinking and their communication of this process as they interacted with the interviewer. Communication is a key aspect of current mathematics instruction (NCTM, 2000) and in the case of students who are less proficient in English it is especially important to think of communication as multifaceted involving gestures, expressions, drawings, and objects as resources to simultaneously communicate mathematical ideas (Moschkovich, 2002). The language structure of assessment items could present added cognitive demands for the students (Campbell, Adams, & Davis, 2007) and hence it is pertinent to take into consideration the students’ multiple resources for communication of their reasoning in solving the problems.

Our interviews uncovered several themes related to students’ thinking. These themes included issues of language, the use of visual cues by students, and mathematical communication.

**Issues of Language**

The wording in the TS problem, “If both the square and the triangle above have the same perimeter, what is the length of each side of the square?” proved to be the major stumbling block for students who got the problem incorrect. We identified several problems that our students had with the wording. One was the use of term “above”; a few ELLs were not sure what that word meant in the context. A second problem was with the word “same” as interpreted as the measure of the length having to be the same (an example of this is shown later in this article), thus choosing (often visually) a length of the triangle that looked like one of the sides of the square. And yet a third issue was with the use of “if … then.” For example, one fourth grade student
interpreted the “if” statement as “they do not have the same perimeter.” When probed, she said “but they do not because it says IF (emphasis added).” This child was interpreting the “if” statement as a negation statement, therefore, the square and the triangle could not possibly have the same perimeter. The interviewer proceeded to say that the perimeter of the triangle and the square were the same; the student then came up with the sides of the square as 1,3,7,9, which added up to 20 but ignored the fact that a square had equal sides. On being asked separately about a square, the student mentioned that it “had four sides.” From the discussions and the students’ representations we concluded that besides the language in the question, the student also required more opportunities understanding the properties of a square.

Yet, in other cases, the wording of the problem was the main stumbling block. As soon as the students understood that the square and triangle had the same perimeter, they could solve the problem. For example, a sixth grade student had difficulty understanding the statement of the problem and asked us for assistance.

[The student added the lengths of the sides of the triangle and worked that out to be 20.]

S₁: I don’t understand it.

I₁: Ok, do you want me to read it or do you want to read it out aloud?

[The student reads the problem]

I₁: Ok do you know what the word perimeter means?

S₁: The outside of umm [points to shapes]

I₁: Ok and so they give you this triangle and give you the measurements of the sides of this triangle and they give you a square and they don’t give you the sides. So they are
telling you that “If the square and the triangle have the same (emphasis) perimeter what is the length of the sides of this square?”

S₁: Five!

I₁: How’d you do that?

S₁: This together is 20 [points to the triangle and the work she did before] and this has 4 sides [pointing to the square] and 5 times 4 is 20.

By emphasizing the word “same,” the interviewer seemed to help the student make the needed connection. When first given to her, this student was not able to solve the problem. If we had not been in this one-to-one interaction, we would not have known if it was because she did not understand the problem or because she did not understand some aspects of the mathematics involved. With the first student (the fourth grader), further questioning uncovered gaps in the students’ understanding of a square and this went beyond the language. Both these examples point to the impact of language on the performance of the students.

**Use of Visual Cues**

Students resorted to the use of grids in finding lengths, area and perimeter. For example, in the TS problem, some students estimated the length of the square by visual approximation, using the lengths of the sides of the triangle as a reference, thus coming up with 4 or 5 as most common answers. When asked about the piece that the two shapes had the same perimeter, some students did not seem to find this information necessary towards answering the problem. Estimating the length could indeed lead to the “right” answer, but what would we know about the student’s understanding of the problem? For example, a fourth grade student drew a 5 x 5 grid (Fig. 2) in the square and concluded that the length of the side of the square had to be 5.
This student stated that he did not need to know the other conditions given in the problem for the solution. When asked if he could have use another grid, he drew a 10 x 10 grid within the same square. This caused a moment of conflict in the students’ thinking, but he still concluded that 5 was the correct choice since none of the other choices were 10. We wonder what he may have done, had we suggested a 4 by 4 grid.

[Figure 2 about here]

**Mathematical Communication**

The task-based interviews provided a strong vehicle for communication with the students about their processes and the mathematics. Within this communication the students were forced to be precise in their responses to the probes by the interviewer. Below a sixth grade student discusses her solution for the TS problem. The students’ communication of her thinking was not clear and she was repeatedly asked to clarify her use for “length”.

[The following episode took place after she worked independently on the TS problem and chose 5 as her solution]

I₂: Ok can you tell me how you came up with 5?

(pause)

S₂: Well umm since the…length of the square this one [points to the triangle] is the same…

I₂: The length of the square is the same as what?

S₂: This one [points to the square]

I₂: This is the square [points to the square] right? (uh-huh) so I um…what do you mean by the length of the square is the same? The same as …as…
S2: No, this is the same [points to the triangle cut-out] as this [points to the square cut-out]

I2: Ok when you say this, you mean the shape?

S2: The length

I2: The length…the length of what? (pause) I mean there is something there that is the same…Yes…but I am not really sure I’m understanding what you are saying.

The student struggled to communicate her thinking as she used ‘length’ instead of ‘perimeter’. By saying that “this [pointing to the triangle] and this [pointing to the square] are the same” it was not clear if the student referred to the lengths of the sides, perimeter or area. The student had marked 5 as her solution and had done no work on the paper. When the interviewer asked her to explain how she decided on 5 (after the exchange above), the student eventually said that she did not remember. The more she tried to explain it, the more confused she seemed to get with both the mathematics (bringing in strategies that are usually associated with finding areas) and the language (having difficulty using precise vocabulary). Successful students were able to explain their reasoning with more linguistic precision than those who struggled, who tended to use multiple pronouns with no clear referents. These findings emphasize the importance of engaging all students, but particularly ELLs, in situations in which they have to communicate their mathematical thinking. Successful students were also proficient in using and going across multiple representations (verbal, diagram, symbolic). For example, in the AC problem, a preferred strategy was to use the cut-outs to fold and “cut” to show that the triangle and the square had the same area. (See figure 3)

[Figure 3 about here]
The interviewer would then often ask if the student could think of a different way to approach the problem. Although many students would make a rectangle with the two squares and another rectangle with the two triangles, and see that they had the same area, they had a hard time using this information to reason about the area of one square and one triangle. In the excerpt below, we see a sixth grader who had already given an argument (by folding and cutting) for why the areas were the same, and is now trying to use the two rectangles (one with the two triangles and one with the two squares) to provide another argument:

S₄: Cause this is half of a rectangle [points to the square cut-out] and this will be half [points to the triangle cut-out] of this rectangle [points to the rectangle formed from the triangles.
I₂: Ah huh
S₄: Like say this rectangle … they both…the area is a 100 and if you cut ‘em in half, this will be 50 [holds up the square cut-out] and this will be 50 [holds up the triangle cut-out]

The student’s use of numbers was effective in communicating his solution to the interviewer. Yet another sixth grader represented his manipulations of the concrete shapes in the AC problem with rectangles and with the equation 2P=2N (“P” represented the triangle and “N” represented the square). He reasoned that if 2P=2N, then half of each rectangle is the triangle P and the square N, therefore, P=N and the areas were equal. These examples illustrate the power of the interview as a technique to find out more about students’ understanding. In these two cases, the students had been successful with their first method, yet by further probing we learn more about the depth of their understanding as they use other approaches to convince the interviewer.
Closing thoughts

The central goal of the interviews is to ensure that the students’ thinking is made explicit so that the interviewer can make interpretations and plan next steps. In order to achieve this goal our experience pointed to three important factors. The first factor related to the choice of tasks for the interview. These tasks had to challenge the students at the appropriate level and also generate sizable student thinking. For example, we observed that the TS problem generated more language issues than the AC problem where the students could mediate their interactions using the cut-outs. There was some degree of experimentation of tasks before we found the appropriate tasks. The declared NAEP questions proved to be a good source of tasks.

The second aspect that we found challenging was determining a line of questioning, usually on-the-fly, if the student generated an unexpected solution. We became better in subsequent interviews as we built on our prior experiences with student responses.

The third aspect that we had to be mindful of during the interviews was the tension between the students getting frustrated, on the one hand, and providing hints that could lead the students towards a solution. Our rapport with the students played a big role in them opening up to us in the interviews and allowing us to ask probing questions. However, we were cautious to cut short the interview if students showed signs of being overwhelmed.

The benefits of doing interviews are valuable in gaining a deep understanding of student thinking, especially when students may feel insecure about participating in whole class mathematical discussions, as may be the case with ELLs. Interviews can provide a safe setting for students to express their mathematical ideas and to practice communicating about mathematics, which may encourage their participation in whole-class discussions. The experiences that students have during the interviews are crucial for students to develop their
mathematical language, build their confidence with mathematical ideas, and further develop their thinking in mathematics. These experiences are especially crucial for Latino/a ELL students because they are developing both their mathematical thinking and their English language in mathematics through these conversations.

In conducting these interviews, teachers not only are able to assess their students’ understanding, but also by seeing the potential obstacles that language creates, they may be able to use this information in their instruction. Interviews could form the base for discussions among teachers at a school as they discuss observations about the students’ thinking and possible instructional strategies to overcome obstacles. Certainly time to conduct these interviews is an issue. Possibly, the teacher could do an interview during students’ independent work time, and maybe interview pairs of students, which also provides for very rich opportunities for students’ communication about mathematics.

References


LIST OF FIGURES

Problem 1: The Triangle and Square problem (TS problem)
If both the square and the triangle above have the same perimeter, what is the length of the side of the square?
   (a) 4 (b) 5 (c) 6 (d) 7 (NAEP, 1996)

Problem 2: The Area Comparison problem (AC problem)
(Cut outs of N and P are given with the base of P being twice the side of the square) Bob, Carmen and Tyler were comparing the areas of N and P. They each conclude the following:
   Bob: N and P have the same area;
   Carmen: The area of N is larger;
   Tyler: The area of P is larger.
Who was correct? Use pictures and words to explain why. (NAEP, 1996)
3. Triangle & Square Problem

If both the square and the triangle above have the same perimeter, what is the length of each side of the square?

A. 4
B. 5
C. 6
D. 7

Figure 2
I put the square into
or the other shape. Since
a saw that little
space was left I put that
space in the square and
saw that N and p were
the same area.