HOW COMPLEX CAN A LAND SUITABILITY MAP BE?

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Abstract

On a land suitability assessment map that is derived from an overlay of \( n \) parent maps, each bearing \( k \) land capability categories, how many permutations of land suitability scores can there be when the scores are calculated by the commonly used rating and weighting method? In this paper, we estimate that this number is between \( k^{\frac{n(n-1)}{2}} \) and \( k^{n+1}/(k^n!k^{n-1}!\ldots k!)^{k-1} \).

1. Introduction

In this article, we aim at answering a basic question in land suitability assessment regarding the relationship between the number of distinct land suitability rankings, that is, permutations of land suitability scores, on a land suitability assessment map and the map’s
structural makeup. More specifically, on a land suitability assessment map that derives from an overlay of \( n \) parent maps, each bearing \( k \) land capability categories, we try to determine how many permutations of land suitability scores there can be when the scores are calculated by the commonly used rating and weighting (RAW) method, which is given by

\[
S = C_{1j_1}w_1 + C_{2j_2}w_2 + \cdots + C_{nj_n}w_n. 
\]  

(1)

In equation (1), for each fixed \( m \), the \( C_{mj} \) is the \( j \)-th land capability score on the \( m \)-th parent map \( (1 \leq m \leq n \) and \( 1 \leq j \leq k \)), and \( w_k \) is the weight of the \( k \)-th parent map \( w_k \geq 0 \) and \( w_1 + w_2 + \cdots + w_n = 1 \).

To illustrate this research question and the pertinent concepts, let’s consider the following example (Diao and Xiang, 2002). On a land suitability assessment map that involves an overlay of 2 parent maps, each bearing 3 land capability categories (that is, \( n = 2 \) and \( k = 3 \)), there are 9 permutations of land suitability scores, that is, 9 unique sets of suitability rankings. Notice that in this case, each \( S \) function is a simple straight line and there are nine such functions. A particular suitability ranking is the same as the order of the \( y \) coordinates of these lines as shown in figure 1 below (Diao and Xiang, 2002, p.675). One can easily see that there are only nice different such orders (marked by the five vertical dotted line segments and the four regions bounded in between).

That number however quickly jumps to 499 when a third parent map with 3 land capability categories (that is, \( n = 3 \) and \( k = 3 \)) is incorporated. As illustrated by figure 2 below (Diao and Xiang, 2002, p.684), it is much harder to count these different rankings. Each \( S \) function is now represented by a plane equation and there are 27 such equations.
2. Choice value set and the number of permutations

The primary motivation of our research is that the knowledge about this relationship is
Figure 2: The different rankings are represented by the bounded regions, their straight boundary edges and their boundary corner points.

essential to the design of choice value set.

The weights $W$ in equation (1) are importance values of the individual parent maps. They are determined on the preferential information solicited from human experts. In almost every elicitation method-ranking, rating, and ratio questioning (Hobbs, 1980, pp.727-728, Hobbs and Meier, 1994, Xiang, 2001, p.62), the integral task of preference elicitation is facilitated by a choice value set. This is a set of discrete or continuous numbers from which a person chooses to express his/her perceptions about a map’s importance or relative importance. In ranking, the choice value set comprises sequential numbers for order designation.
That is, 1 represents the highest level of importance, 2 second, 3 third, etc. In rating, the choice value set can either be pre-defined for the users, containing a finite number of choice values usually with a constant increment, or defined by users who may select any values at any intervals, constant or variable. Table 1 provides examples of choice value sets and their theoretical value ranges. In ratio questioning, the choice value set usually contains integers indicating the relative importance between a pair of maps. The most commonly used is Satty’s set of \{1, 2, 3, 4, 5, 6, 7, 8, 9\} in which 1 suggests the pair of maps be "equally important", 2 one map be "slightly more important" than the other, 3 "moderately more important", etc. (Satty, 1980, Xiang and Whitley, 1994).

Recent studies have shown that these choice value sets suffer from the problems of redundancy and under-representation, which can significantly affect the quality of map overlays and land suitability assessment (Xiang, 2000, Xiang and Salmon, 2001, Diao and Xiang, 2002). Through a land suitability assessment for park-site selection, Xiang and Salmon (2001) showed that some numerical values in such choice value sets are equivalent to one another functionally because they produce overlaid maps that share an identical site ranking, that is, a same permutation of land suitability scores. Should the site selection be based solely on the rankings, which is often a common practice in real world decision-making (Holmes, 1972, 1973, Kirkwood and Corner, 1993, Srivastava et al, 1995, Barron and Barrett, 1996), the use of any one of these maps will lead to virtually the same decision. The inclusion of these functionally equivalent numerical values is therefore redundant. Ironically, as Xiang and Salmon further demonstrated, the very same choice value sets are also incomplete for they represent only a partial array of preferential choices. A number of overlaid maps that have unique permutations are concealed and placed out of a user’s sight simply because their corresponding weight values are not represented in the choice value
set. As the use of choice value sets is a way, and in many cases the only way, to express perceptions about the importance or relative importance of maps, a user would have made different site selections if he or she were presented with the complete array of preferential choices. From this perspective, the redundant choice value set is also inadequate and under-representative.

To avoid these problems, ideally, a choice value set should be designed with permutations (Xiang and Salmon, 2001, Diao and Xiang, 2002). That is, each number in an ideal choice value set represents a unique permutation of land suitability scores, and all the permutations on a land suitability assessment map are represented by the choice value set. The knowledge about the relationship between the number of permutations on a land suitability map and the map’s structural makeup is therefore essential to the design of such an ideal choice value set.

The second motivation of our investigation is that after an intensive literature search, we found that little if any work has been reportedly done on this relationship. The literature on map overlays falls into two general categories. Studies under the first category are focused on the relationship between the number of areas on an overlaid map and the numbers of areas on its parent maps, and best characterized by the question raised by Saalfeld (1991, p.23) “If I overlay a map of \( n \) regions on another map of \( m \) regions, how many regions can there be in the composite map?” The second category of work on map overlays is concentrated on the accuracy issues. Following MacDougall’s pioneering work in 1975 (MacDougall, 1975), this stream of research has produced a larger quantity of reports than the first category with various issues on errors in map overlays in both vector and raster environments. Examples include, but are not limited to, Chrisman (1987), Veregin and Lanter (1995), and Agumya and Hunter (2002). In neither category of studies, however, the relationship under our
investigation has ever been investigated. The state of underdevelopment, we believe, is mainly due to the extreme difficulty of finding an equation that can accurately predict the magnitude of this relationship with any combination of $n$ (the number of parent maps) and $k$ (the number of categories in each map). As a first step toward a thorough understanding of this relationship, in the following two sections, we will discuss the lower and upper bounds on the total number of possible site rankings, that is, the range of the number of possible permutations, on a land suitability assessment map with $n$ parent maps each bearing $k$ categories.

3. The lower bound

We will assume a general set up of the site ranking problem under the overlaid map setting. That is, we would have $n$ parent maps with each map having $k$ land capability categories. Let $C_{ij}$ be the value of the $j$-th category of the $i$-th parent map, where $0 \leq C_{ij} \leq 1$, with $C_{i1} = 0$ being the least capable and $C_{ik} = 1$ being the most capable. It is a common practice that the values of $C_{ij}$ are evenly spaced on the interval $[0, 1]$. For example, in the case of $k = 3$, one would simply choose $C_{i1} = 0$, $C_{i2} = 0.5$ and $C_{i3} = 1$ and in the case of $k = 4$, one would choose $C_{i1} = 0$, $C_{i2} = 1/3$, $C_{i3} = 2/3$ and $C_{i4} = 1$.

Definition 1 A vector $\vec{W} = (w_1, w_2, ..., w_n)$ is called a weight value vector of the $n$ overlaid parent map’s weights if it satisfies the condition $w_j \geq 0$ and $w_1 + w_2 + \cdots + w_n = 1$.

For each set of the constants $\{C_{ij}\}$ ($i = 1$ to $n$ and each $j_i$ is between 1 and $k$), the suitability function $S$ is defined in (1) as

$$S = C_{1j_1}w_1 + C_{2j_2}w_2 + \cdots + C_{nj_n}w_n,$$
where \((w_1, w_2, ..., w_n)\) is a weight value vector of the parent maps.

Notice that there are \(N = k^n\) such suitability functions. For example, if \(n = k = 3\), then there are 27 suitability functions. For each fixed weight choice \(W = (w_1, w_2, ..., w_n)\), the values of the \(k^n\) suitability functions determine an order among them. This basically means re-arranging the order of \(S_1, S_2, ..., S_N\), or, a permutation of \(N\) letters. In other word, the original functions \(S_1, S_2, ..., S_N\) so indexed correspond to the \(N\) letter \((1, 2, ..., N)\) and the ranking order changes that into \((\pi(1), \pi(2), ..., \pi(N))\) where \(\pi\) is a permutation (re-arrangement) of the \(N\) letters. To stress the fact that this permutation is determined by the weight vector \(\vec{w} = (w_1, w_2, ..., w_n)\), we may denote the permutation by \(\pi_{\vec{w}}\). In the case two capability functions have the same value at the given \(\vec{w}\), their orders in the permutation become a problem. For instance, if there are three capability functions and their values at the given \(\vec{w}\) are \(S_2 > S_1 = S_3\), then the permutation could be either \((2, 1, 3)\) or \((2, 3, 1)\). If no two capability functions share the same value at the given \(\vec{w}\), then there is no confusion in the definition of \(\pi_{\vec{w}}\) and in this case we will call the corresponding ranking given by \(\pi_{\vec{w}}\) a proper ranking. Following Diao and Xiang, (2002), we will call the set of all weight vectors \(\vec{w} = (w_1, w_2, ..., w_n)\) that produce the same ranking (i.e., the same permutation) a ranking segment. In particular, a ranking segment whose corresponding ranking is proper is called a type A segment. Notice that the number of distinct segments equals the number of all possible rankings of the capability functions. Each ranking can be represented by a weight vector from the corresponding segment. It can be shown that type A segments are the interiors of some convex subsets in the \((n - 1)\)-th dimensional space and the other segments are the boundaries of these convex subsets at different dimensions. The case of \(n = 3\) is illustrated in figure 1. In this section, we will focus our investigation on the proper rankings for two reasons. Firstly, this avoids the confusion with regard to the permutations and simplifies the problem mathematically. Secondly, since each representation of a segment
of an improper ranking is on the boundary of a segment representing a proper ranking, and can thus be approximated by a proper ranking closely, ignoring it will not cause a big problem in practice.

In a weight value set $V$, each element is a weight vector representing certain segment. The set $V$ is a redundant set if there exist two elements representing the same segment, and it is an under representation set if some segments are not represented by any weight vector in the set. $V$ is called an ideal weight value set if it contains a representative for each segment and is redundancy free. Although the examples in Diao and Xiang, (2002) shows that this number is increasing rapidly, there is no known formula for the exact number of segments. In fact, there is not even an estimate about this number in the literature. In the following theorem, we derive a lower bound on the number of type A segments.

**Theorem 1** Assume that there are $n$ maps and each map has $k$ (equally spaced) categories (equally spaced over $[0, 1]$), then the total number of type A segments (denoted by $A(n)$) is bounded below by $k^{\frac{n(n-1)}{2}}$.

For example, if there are three parent maps each with three capability categories, then there are at least $3^3 = 27$ type A segments (there are actually 156 of them by Diao and Xiang (2002), and if there are 5 maps with three categories each, then there are at least $3^{10} = 59049$ type A segments.

We will prove the theorem by first establishing a recursive relation between $A(n)$ and $A(n + 1)$, the numbers of type A segments in the case of $n$ and $n + 1$ parent maps (under the assumption that each map has $k$ equally spaced categories).

Let $\hat{W} = (w_1, w_2, ..., w_n)$ be a weight value vector from a type A segment. Let $S_1$,
$S_2$, ... , $S_N$ (where $N = k^n$ is the total number of suitability functions) be the suitability functions, indexed according to their rankings at $\vec{W}$. That is, $0 = S_1(\vec{W}) < S_2(\vec{W}) < \cdots < S_{N-1}(\vec{W}) < S_N(\vec{W}) = 1$. That the values of $S_1$, $S_2$, ... , $S_N$ at $\vec{W}$ are all distinct is by the definition of type A segment. If we plot these values on the $S$-line, we will get a set of points within the range of 0 to 1. This is illustrated in figure 3 in the case of $N = 3^2 = 9$. Notice that $S_N = 1$ corresponds to the case of highest category in each parent map and $S_1 = 0$ corresponds to the case of lowest category in each parent map.

Figure 3: The ranking of the suitability functions $S_i$'s determined by their values at $\vec{W}$ in the case of $n = 2$ and $k = 3$.

we will now examine how many different type A segments we may produce using the weight vector $\vec{W}$ above in the following way.

Let $w_{n+1}$ be a number between 0 and 1 and consider the new weight vector $\vec{W}' = (w'_1, w'_2, ..., w'_n, w'_{n+1})$, where $w'_1 = w_1 w_{n+1}$, $w'_2 = w_2 w_{n+1}$, ... , $w'_n = w_n w_{n+1}$ and $w'_{n+1} = 1 - w_{n+1}$. We will leave it to our reader to verify that $\vec{W}'$ is a weight value vector of $n + 1$ parent maps.

Let us first consider the case when $w_{n+1}$ is fixed at a small value, say it is less than $\frac{1}{k}$. Notice that now there are $N_1 = k^{n+1} = k \cdot k^n = k \cdot N$ suitability functions involved.

We will index them as $S_{(i,j)}$ so that when they are evaluated at $\vec{W}'$, we have $S_{(i,j)} = w_{n+1} \cdot S_i + (1 - w_{n+1})C_{tj}$ where $t = n + 1$ (this is simply for the sake of simplicity in the notation), $1 \leq j \leq k$ and $C_{tj}$ is the $j$-th category score of the $(n + 1)$-th parent map.

Recall that $j = 1$ corresponds to the lowest category of the $(n + 1)$-th parent map, that is,
$C_{t1} = 0$. So the ranking of the suitability functions $S_{(i,1)}$ at $\vec{W}'$ is determined by the values $S_{(i,1)}(\vec{W}') = w_{n+1} \cdot S_i(\vec{W}) + (1 - w_{n+1})C_{tj} = W_{n+1} \cdot S_i(\vec{W})$ $(1 \leq i \leq N)$, which is the same as that determined by the $S_i(\vec{W})$’s. The only difference is that the values of the $S_{(i,1)}(\vec{W}')$’s are now bounded between 0 and $W_{n+1}$. Since $w_{n+1} < \frac{1}{k}$, this implies that the values of the $S_{(i,1)}(\vec{W}')$’s are now bounded between 0 and $\frac{1}{k}$. Now consider the case $j = 2$, that is, the second lowest category of the $(n+1)$-th parent map. By our assumption, $C_{t2} = \frac{1}{k-1}$. In the following, we will show that the range of the values of the suitability functions $S_{(i,2)}(\vec{W}')$ is between $\frac{1}{k}$ and $\frac{2}{k}$.

We know that $S_1 = 0$ is the lowest value we have for all the $S_i$’s. So the lowest value of the $S_{(i,2)}(\vec{W}')$’s is $S_{(1,2)}(\vec{W}') = w_{n+1} \cdot S_1 + (1 - w_{n+1})C_{t2} = (1 - w_{n+1}) \frac{1}{k-1}$. Since $w_{n+1} < \frac{1}{k}$, $1 - w_{n+1} > 1 - \frac{1}{k} = \frac{k-1}{k}$. So $1 - w_{n+1} \frac{1}{k-1} > \frac{k-1}{k} \cdot \frac{1}{k-1} = \frac{1}{k}$. On the other hand, the largest value the $S_i$ can have is $S_N = 1$. So the largest value of the $S_{(i,2)}(\vec{W}')$’s is $S_{(N,2)}(\vec{W}') = w_{n+1} \cdot S_N + (1 - w_{n+1})C_{t2} = w_{n+1} + (1 - w_{n+1}) \frac{1}{k-1}$. We have

$$w_{n+1} + (1 - w_{n+1}) \frac{1}{k-1} = \frac{1 + (k - 2)w_{n+1}}{k-1}$$

$$< \frac{1 + (k - 2) \frac{1}{k}}{k-1} = \frac{2(k-1)}{k(k-1)} = \frac{2}{k}.$$

This shows that the range of values of the suitability functions $S_{(i,2)}$ at $\vec{W}'$ is between $\frac{1}{k}$ and $\frac{2}{k}$. Which implies that the values of the suitability functions $S_{(i,2)}$’s are larger than those of the suitability functions $S_{(i,1)}$’s when they are evaluated at $\vec{W}'$.

Similarly, we can prove that the range of the values of the suitability functions $S_{(i,j)}$ at $\vec{W}'$ is between $\frac{j-1}{k}$ and $\frac{j}{k}$. It follows that all the suitability function values are distinct at $\vec{W}'$. This situation is illustrated in figure 4 for the case of $k = 3$, $N = 2$ and $N + 1 = 3$. 

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Figure 4: The values of suitability functions $S_{ij}$ at $\vec{W}'$ are all distinct and are divided into groups according to the index $j$.

Next we prove that there are only finitely many values of $w_{n+1}$ (in the range of 0 to 1) that can produce an improper ranking of the $S_{ij}$ at $\vec{W}'$. Let us remind our reader that so far we have kept the vector $\vec{W}$ fixed, so the only variable we have is $w_{n+1}$ at this point. Observe that when we have an improper ranking, we must have

$$S_{i_1j_1}(\vec{W}') = S_{i_2j_2}(\vec{W}')$$

for two distinct pairs $(i_1, j_1)$ and $(i_2, j_2)$. This leads to the following equation about $w_{n+1}$:

$$w_{n+1}((S_{i_1}(\vec{W}) - S_{i_2}(\vec{W})) - (C_{tj_1} - C_{tj_2})) = C_{tj_2} - C_{tj_1}.$$  

If this equation is to have more than one solution, then we must have $C_{tj_2} - C_{tj_1} = 0$, which implies that $j_1 = j_2$. Thus we must have $i_1 \neq i_2$ since $(i_1, j_1) \neq (i_2, j_2)$. Therefore $S_{i_1}(\vec{W}) \neq S_{i_2}(\vec{W})$ and the only solution to the above equation is $w_{n+1} = 0$, which is a contradiction. Since there are only finitely many equations of the form 2 and each one can only have at most one solution, there are only finitely many values of $w_{n+1}$ that may produce improper rankings of the $S_{ij}$’s.

We now consider the other extreme case of $w_{n+1}$, namely when it is very close to 1, say $w_{n+1} > \frac{1}{k(w_{n+1})_{S_2}}$. (Recall that $S_2(\vec{W}) > 0$ since $S_1(\vec{W}) = 0$ and all the $S_i(\vec{W})$’s are distinct.) We will leave it to our reader to verify that under this condition, we have

$$S_{21}(\vec{W}') = w_{n+1}S_2(\vec{W}) > S_{12}(\vec{W}') = \frac{k}{k-1} - w_{n+1}.$$
Since we have $S_{i1}(\vec{W}') > S_{21}(\vec{W}')$ for all $i > 2$ (regardless what $w_{n+1}$ is), this implies that $S_{i1}(\vec{W}') > S_{12}(\vec{W}')$ for all $i \geq 2$. So, by a continuity argument, as $w_{n+1}$ changes from 0 to 1, there exactly $N - 1$ values of $w_{n+1}$ such that at each of them, the values of some $S_{i1}(\vec{W}')$ and $S_{12}(\vec{W}')$ switch order, producing a new proper ranking immediately after the switch (before it hits another switching point for any other pairs). So there will be at least $N$ proper rankings (the original one when $w_{n+1}$ is very small and the $N - 1$ that we pick up after the $N - 1$ switches) for the $S_{ij}$’s when determined by the vector $\vec{W}'$. Finally, observe that if we now choose the $\vec{W}$ from a different type A segment, then we will produce $k$ totally new proper rankings following the same argument above. In other word, each type A segment in the case of $n$ parent maps will lead to at least $N = k^n$ distinct type A segments in the case of $n + 1$ parent maps. That is $A(n + 1) \geq k^n A(n)$. Solving this recursive inequality leads to our conclusion.

4. The upper bound

In this section, we will briefly discuss the issue regarding an upper bound of the number of all possible rankings under the assumption that there are $n$ parent maps each with $k$ categories. As it is unlikely an explicit formula can be derived at this time, a good estimate of this upper bound would also be helpful for us to understand the complexity of the problem. However, it turns out that the method we used in last section does not apply to this problem and we have not been able to derive a reasonably small upper bound.

Under the general set up of the site ranking problem of the overlaid maps given in the last section, we see that if there are $n$ parent maps each with $k$ categories, then we would
have \( N = k^n \) suitability functions and each individual ranking is a permutation of these functions. One could get a very rough (and obvious) estimate of the total number of these rankings by assuming that all possible rankings could be attained by the suitability functions at some weight value. That is, the total number of these rankings is bounded above by \( k^n! \). However, this bound is so large that it could not give us anything meaningful. For example, for \( n = 2 \) and \( k = 3 \), this bound is \( 9! = 362,880 \) and the actually total number is a mere 9. For \( n = 3 \) and \( k = 3 \), this bound is \( 27! \), which is more than \( 10^{28}! \) (Compare this with our known number of 499.) One can improve this estimate somewhat by the following approach. Recall that for \( N = k^n \), we may index the suitability functions as \( S_1, S_2, ..., S_N \). Then for the case of \( n + 1 \) parent maps each with \( k \) categories, the suitability functions can be indexed as \( S_{(i,j)} \) where

\[
S_{(i,j)} = w_{n+1} \cdot S_i(\vec{W}_n) + (1 - w_{n+1})C_{tj}
\]

where \( t = n + 1, 0 \leq w_{n+1} \leq 1, \vec{W}_n \) is a weight vector of \( n \) entries and \( C_{tj} \) is the \( j \)-th category score of the \( (n + 1) \)-th parent map. For any given ranking of these \( k^{n+1} = kN \) suitability functions (determined by some weight vector \( \vec{W}_{n+1} \), if we ignore the contribution from the last parent map, then we will obtain a ranking of the suitability functions \( S_1, S_2, ..., S_N \). This observation tells us that each ranking of the functions \( S_{ij} \) can be reconstructed from a ranking of the functions \( S_i \). So the answer to the following question will give us an upper bound on the total number of distinct rankings in the case of \( n + 1 \) parent maps in terms of the number of total rankings in the case of \( n \) parent maps:

How many different rankings among the functions \( S_{ij} \) may we have using a given ranking in the functions \( S_i \)?

Observe that for each given \( C_{tj} \) and \( \vec{W}_{n+1} = ((1 - w_{n+1})\vec{W}_n, w_{n+1}) \), the ranking of the
functions

\[ S_{(i,j)} = w_{n+1} \cdot S_i(\bar{W}_n) + (1 - w_{n+1})C_{tj} \]

is the same as that of the original \( S_i(\bar{W}_n) \)'s. There are \( k \) such groups of suitability functions (one group for each fixed \( j \)). Therefore, there can be at most \( (kN)!/(N!)^k = k^{n+1}/(k^n!)^k \) distinct rankings among the functions \( S_{(i,j)} \). In other word, if \( B(n) \) is the total number of rankings for the case of \( n \) parent maps and \( B(n+1) \) is the total number of rankings for the case of \( n + 1 \) parent maps, then we have

\[ B(n + 1) \leq \frac{k^{n+1}}{(k^n!)^k} B(n). \]

This gives a slightly better upper bound than we had earlier. For example, since we know that \( B(2) = 9 \) (assuming that \( k = 3 \) as we did before), the above estimate would give us

\[ B(3) \leq \frac{3^{3!}}{(3^2!)^3} B(2) \approx 2.05 \times 10^{12}. \]

Although this is much smaller than \( 10^{28} \), it is still too big to be of practical use. This is a wide open area for us to explore further.

5. Conclusions

The lower and upper bounds of the number of possible rankings provide useful information for us to understand the complexity of the ranking problem in the overlaid map settings, especially for the design of choice value sets. While it is possible to develop softwares to tabulate all these possible rankings, the lower bound we found in Section 3 suggests that it is not practical for large \( n \) and \( k \) in general since the amount of computation is too large. This also suggests the need of using a choice value set that is smaller in size. However, in order for a choice value set to represent the rankings in an effective and meaningful way,
we will need to have a good understanding of the structures of the possible rankings, not just the number of them. This work is only a first step in achieving that goal.

References


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</tbody>
</table>

Table 1: Examples of weight value sets in rating methods (after Xiang 2000, p.600). Where $\sum S_j$ is the sum of the suitability scores and $\sum w_i$ is the sum of weights.