The Linearity of the Ropelengths of Conway Algebraic Knots in Terms of Their Crossing Numbers

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Abstract

For a knot or link $K$, let $L(K)$ denote the ropelength of $K$ and let $Cr(K)$ denote the crossing number of $K$. An important problem in geometric knot theory concerns the relationship between $L(K)$ and $Cr(K)$ (or intuitively, the relationship between the length of a rope needed to tie a particular knot and the complexity of the knot). We show that there exists a constant $a > 0$ such that if a knot $K$ allows a special knot diagram $D$ (called Conway algebraic knot diagram) with $n$ crossings, then $L(K) \leq a \cdot n$. Furthermore, if $D$ is alternating (but not necessarily reduced and in fact $K$ may not have a minimal alternating diagram that is algebraic), then $L(K) \leq a \cdot Cr(K)$. The approach used here can be applied to a larger class of knots, namely those formed by replacing single crossings in a Conway algebraic knot diagram by tangles whose crossing number is bounded by a constant. Interestingly, it has been shown by the same authors that the Jones polynomials of these knots can be computed in polynomial time.

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