Abstract

The deficiency of a link $K$ is defined as $d(K) = Cr(K) - 2g(K) - b(K) - \mu(K) + 2$, where $Cr(K)$ is the crossing number of $K$, $g(K)$ is the genus of $K$, $b(K)$ is the braid index of $K$ and $\mu(K)$ is the number of components of $K$. It is known that $d(K) \geq 0$ for any $K$ hence $Cr(K)$ is bounded below by $2g(K) + b(K) + \mu(K) - 2$. It is known that the crossing numbers of deficiency zero links are additive under the connected sum operation. A main result in this paper is a proof that shows the number of deficiency zero links grows exponentially with respect to the crossing numbers of the links. Another part of the paper concerns the problem of whether the braid index and genus of a link can be combined to give an upper bound of the crossing number of the link. We conjecture that for any link $K$, we have $Cr(K) \leq 2(b(K) + g(K) + \mu(K))$. Using the help of the HOMFLY polynomial, we have verified this inequality for links up to 12 crossings. On the other hand, we also prove that this inequality is indeed true for all two-bridge links. In fact, we prove that if $K$ is a two-bridge link, then $Cr(K) \leq 2g(K) + 2b(K) + \mu(K) - 4$, which is a stronger result than the conjecture.