ORTHOGONAL POLYNOMIALS REPRESENTED

BY CW-SPHERES

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Abstract

Given a sequence \( \{Q_n(x)\}_{n=0}^\infty \) of symmetric orthogonal polynomials, defined by a recurrence formula \( Q_n(x) = \nu_n \cdot x \cdot Q_{n-1}(x) - (\nu_n - 1) \cdot Q_{n-2}(x) \) with integer \( \nu_i \)'s satisfying \( \nu_i \geq 2 \), we construct a sequence of nested Eulerian posets whose ce-index is a non-commutative generalization of these polynomials. Using spherical shellings and direct calculations of the cd-coefficients of the associated Eulerian posets we obtain two new proofs for a bound on the true interval of orthogonality of \( \{Q_n(x)\}_{n=0}^\infty \). Either argument can replace the use of the theory of chain sequences. Our cd-index calculations allow to represent the orthogonal polynomials as an explicit positive combination of terms of the form \( x^{n-2r}(x^2 - 1)^r \). Both proofs may be extended to the case the \( \nu_i \)'s are not integers and the second proof is still valid when only \( \nu_i > 1 \) s required. The construction provides a new “limited testing ground” for Stanley’s non-negativity conjecture for Gorenstein* posets, and suggests the existence of strong links between the theory of orthogonal polynomials and flag-enumeration in Eulerian posets.

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