Tchebyshev posets

Gábor Hetyei

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Abstract

We construct for each $n$ an Eulerian partially ordered set $T_n$ of rank $n + 1$ whose $ce$-index provides a non-commutative generalization of the $n$-th Tchebyshev polynomial. We show that the order complex of each $T_n$ is shellable, homeomorphic to a sphere, and that its face numbers minimize the expression $\max_{|x| \leq 1} \left| \sum_{j=0}^{n} (f_{j-1}/f_{n-1}) \cdot 2^{-j} \cdot (x - 1)^j \right|$ among the $f$-vectors of all $(n - 1)$-dimensional simplicial complexes. The duals of the posets constructed have a recursive structure similar to face lattices of simplices or cubes, offering the study of a new special class of Eulerian partially ordered sets to test the validity of Stanley’s conjecture on the non-negativity of the $cd$-index of all Gorenstein* posets.

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Department of Mathematics, UNC Charlotte, Charlotte, NC 28223