Realizable Powers of Ropelengths by Nontrivial Knot Families

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Preprint no. 2003-04

Abstract

For any given knot $K$, a thick realization $K_0$ of a knot type $K$ is a knot of unit thickness which is of the knot type $K$. The ropelength of $K$ is defined as the arc length of the shortest thick realization of $K$. A recent result shows that there exists a constant $b > 0$ such that for any knot type $K$, its ropelength $L(K)$ is bounded above by $b \cdot (Cr(K))^{3/2}$, where $Cr(K)$ is the crossing number of $K$. It is also known that there exists a family of infinitely many knot types $\{K_n\}$ such that $n = Cr(K_n) \to \infty$ as $n \to \infty$ and $L(K_n) = O(n)$. In this paper, we show that for each $p$ with $3/4 \leq p \leq 1$, there exists a family of infinitely many knot types $\{K_n\}$ with the property that $Cr(K_n) \to \infty$ (as $n \to \infty$) such that $a_0 \cdot (Cr(K_n))^p \leq L(K_n) \leq b_0 \cdot (Cr(K_n))^p$, where $a_0$ and $b_0$ are some positive constants. In other word, any power between $3/4$ and 1 is realizable by some knot family.

1991 AMS Subject Classification: Primary: 57M25

Key words and phrases: Knots, links, crossing number, thickness of knots, rope length of knots.