The s-elementary Frame Wavelets are Path Connected

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Abstract

An s-elementary frame wavelet is a function $\psi \in L^2(\mathbb{R})$ which is a frame wavelet and is defined by a Lebesgue measurable set $E \subset \mathbb{R}$ such that $\hat{\psi} = \frac{1}{\sqrt{2\pi}} \chi_E$. In this paper we prove that the family of s-elementary frame wavelets is a path-connected set in the $L^2(\mathbb{R})$-norm. This result also holds for s-elementary $A$-dilation frame wavelets in $L^2(\mathbb{R}^d)$ in general. We also consider the uniform path-connectivity of the sets of frame wavelets, normalized tight frame wavelets and s-elementary frame wavelets. We prove that none of these sets is uniformly path-connected.

Key words and phrases: Frames, Wavelets, Frame Wavelets, Frame Wavelet Sets, Fourier Transform.