The Existence of Subspace Wavelet Sets

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Abstract

Let $\mathcal{H}$ be a reducing subspace of $L^2(\mathbb{R}^d)$, that is, a closed subspace of $L^2(\mathbb{R}^d)$ with the property that $f(A^m t - \ell) \in \mathcal{H}$ for any $f \in \mathcal{H}$, $m \in \mathbb{Z}$ and $\ell \in \mathbb{Z}^d$, where $A$ is a $d \times d$ expansive matrix. It is known that $\mathcal{H}$ is a reducing subspace if and only if there exists a measurable subset $M$ of $\mathbb{R}^d$ such that $A^t M = M$ and $\mathcal{F}(\mathcal{H}) = L^2(\mathbb{R}^d) \cdot \chi_M$. Under some given conditions of $M$, it is known that there exist $A$-dilation subspace wavelet sets with respect to $\mathcal{H}$. In this paper, we prove that this holds in general.

Key words and phrases: Frame, Wavelet, Frame Wavelet, Frame Wavelet Set, Fourier Transform.