1. Evaluate the limit: \( \lim_{{x \to 2}} \frac{x^2 - 4}{x^2 + 4x - 12} \).

(a) 0  
(b) 0.25  
(c) 0.5  
(d) 1  
(e) Does not exist

2. Which of the following is the derivative of \( g(x) = x^2 \cos(3x + 1) \)?

(a) \( 2x \sin(3x + 1) \)  
(b) \( -6x \sin(3x + 1) \)  
(c) \( 2x \cos(3x + 1) - x^2 \sin(3x + 1) \)  
(d) \( 2x \cos(3x + 1) + 3x^2 \sin(3x + 1) \)  
(e) \( 2x \cos(3x + 1) - 3x^2 \sin(3x + 1) \)

3. Find an equation for the line that is tangent to the graph of \( f(x) = x^3 - 7x + 4 \) at \( x = 2 \).

(a) \( y = -7x + 4 \)  
(b) \( y = -7x + 12 \)  
(c) \( y = -3x + 4 \)  
(d) \( y = 5x - 12 \)  
(e) \( y = 5x + 4 \)

4. For what values of \( x \), if any, does the function \( f(x) = 3x^4 - 32x^3 + 72x^2 + 10 \) have a local minimum?

(a) There is no local minimum  
(b) Only at \( x = 0 \)  
(c) Only at \( x = 2 \)  
(d) Only at \( x = 6 \)  
(e) At \( x = 0 \) and at \( x = 6 \)
5. Which of the following is the derivative of \( f(x) = e^{3x^2 + 1} \) ?

(a) \( 6x e^{3x^2 + 1} \)
(b) \( e^{3x^2 + 1} \)
(c) \( 6 e^{3x^2} \)
(d) \( e^{6x} \)
(e) \( (3x^2 + 1)e^{3x^2} \)

6. Which of the following is the derivative of \( f(x) = \frac{x^3}{\sin(5x)} \) ?

(a) \( \frac{3x^2}{\cos(5x)} \)
(b) \( \frac{3x^2}{5 \cos(5x)} \)
(c) \( \frac{3x^2 \sin(5x) - 5x^3 \cos(5x)}{\sin^2(5x)} \)
(d) \( \frac{3x^2 \sin(5x) + 5x^3 \cos(x)}{\sin^2(5x)} \)
(e) \( \frac{5x^3 \cos(5x) - 3x^2 \sin(5x)}{\sin^2(5x)} \)

7. A particle is traveling around the circle \( x^2 + y^2 = 25 \) where \( x \) and \( y \) are measured in inches. At the instant the particle is at the point \( (3, 4) \), \( \frac{dy}{dt} = 15 \text{ in/sec} \). Find \( \frac{dx}{dt} \) at this time.

(a) \( -20 \text{ in/sec} \)
(b) \( -15 \text{ in/sec} \)
(c) \( -2.5 \text{ in/sec} \)
(d) \( 15 \text{ in/sec} \)
(e) \( 20 \text{ in/sec} \)

8. Which of the following is the derivative of \( f(x) = \tan(x) \) ?

(a) \( -\cot(x) \)
(b) \( \cot(x) \)
(c) \( \sec(x) \)
(d) \( \sec^2(x) \)
(e) \( \sec(x) \tan(x) \)

9. Which of the following is the slope of the line tangent to the curve \( y^2 + 3x^2 + xy = 36 \) at \( (2, 4) \)?

(a) \( -4 \)
(b) \( -2 \)
(c) \( -1.6 \)
(d) \( -4/3 \)
(e) \( 8/3 \)
10. The derivative of the function \( f(x) \) is given by \( f'(x) = 20x + 6x^{3/2} \). Find a formula for the function \( f(x) \) given that \( f(1) = 25 \).

(a) \( f(x) = 10x^2 + 4x^{3/2} + 11 \)
(b) \( f(x) = 20x^2 + 6x^{3/2} - 1 \)
(c) \( f(x) = 40x^2 + 6x^{3/2} - 21 \)
(d) \( f(x) = 40x^2 + 9x^{3/2} - 24 \)
(e) \( f(x) = 3x^{-1/2} + 22 \)

11. Evaluate the limit: \( \lim_{x \to 2} \frac{21x + 2}{7x - 4} \).

(a) 0
(b) 3
(c) 4.4
(d) 8
(e) Does not exist

12. Which of the following is the derivative of \( f(x) = \ln(8x + 3) \)?

(a) \( \frac{1}{8x + 3} \)
(b) \( \frac{8}{8x + 3} \)
(c) \( \frac{-64}{(8x + 3)^2} \)
(d) \( \frac{-8}{(8x + 3)^2} \)
(e) \( \frac{-1}{(8x + 3)^2} \)

13. Evaluate the limit: \( \lim_{x \to +\infty} \frac{9x + 3e^{-x}}{2x - 5e^{-x}} \).

(a) \(-4\)
(b) 0
(c) 6/7
(d) 4.5
(e) Does not exist
1. Determine the values of $A$ and $B$ (if they exist) using the graph of $f(x)$.

\[
\lim_{x \to -3} f(x) = A \quad \lim_{x \to 2} f(x) = B
\]

(a) $A$ does not exist, $B = -2$
(b) $A = -1$, $B = -2$
(c) $A = -1$, $B = 3$
(d) $A = 2$, $B = -2$
(e) $A = 2$, $B = 3$

2. A cube is measured to have edges of length 20 cm with a possible error no worse than ±0.03 cm. Use differentials to estimate the maximum error in calculating the volume.

(a) ±60.0 cm$^3$
(b) ±36.0 cm$^3$
(c) ±24.0 cm$^3$
(d) ±1.8 cm$^3$
(e) ±0.6 cm$^3$

3. The function $f(x)$ has a derivative for each value of $x$ and $g(x) = \sqrt{f(x)}$. Find $g'(2)$ given that $f(2) = 25$ and $f'(2) = 16$.

(a) $g'(2) = 3.2$
(b) $g'(2) = 1.6$
(c) $g'(2) = 0.2$
(d) $g'(2) = 0.1$
(e) $g'(2)$ does not exist

4. Use Newton’s method to approximate where $f(x) = x^3 + x^2 + 2x + 3$ has a zero. Start with $x_1 = 1$ as the first approximation and calculate $x_2$ and $x_3$.

(a) $x_2 = 0$ and $x_3 = -1.5$
(b) $x_2 = 0$ and $x_3 = -2/3$
(c) $x_2 = 0$ and $x_3 = 1.5$
(d) $x_2 = 2$ and $x_3 = 55/18$
(e) $x_2 = 2$ and $x_3 = 56/19$
5. The graph at right is the graph of the derivative of the function $f(x)$ [so the graph of $y = f'(x)$]. Which of the following statements is true about the function $f(x)$.

(a) $f(x)$ is increasing when $A < x < C$ and $F < x < +\infty$ and concave up when $-\infty < x < B$ and $E < x < +\infty$

(b) $f(x)$ is increasing when $A < x < C$ and $F < x < +\infty$ and concave up when $D < x < +\infty$

(c) $f(x)$ is increasing when $-\infty < x < B$ and $E < x < +\infty$ and concave up when $D < x < +\infty$

(d) $f(x)$ is increasing when $-\infty < x < B$ and $E < x < +\infty$ and concave up when $A < x < B$ and $F < x < +\infty$

(e) $f(x)$ is increasing when $-\infty < x < B$ and $E < x < +\infty$ and concave up when $-\infty < x < D$

6. Which of the following limits represents the derivative of $f(x) = \cos(3x + 1)$?

(a) $\lim_{h \to 0} \frac{\cos(3x + h + 1) - \cos(3x + 1)}{h}$

(b) $\lim_{h \to 0} \frac{\cos(3x + 3h + 3) - \cos(3x + 1)}{h}$

(c) $\lim_{h \to 0} \frac{\cos(3x + 3h + 1) - \cos(3x + 1)}{h}$

(d) $\lim_{h \to 0} \frac{3\cos(x + h + 1/3) - 3\cos(x + 1/3)}{h}$

(e) $\lim_{h \to 0} \frac{\cos(3x + h + 1)}{h}$

7. The derivative of a function $g(x)$ is given by $g'(x) = -7(x + 3)^2(x - 1)(x - 5)$. Find the $x$-coordinates [only the $x$ since you don’t know what $g(x)$ is] for each local maximum and each local minimum of $g(x)$, if any.

(a) Local maxima at $x = 1$ and $x = 5$, local minimum at $x = -3$

(b) Local maximum at $x = 1$, local minimum at $x = 5$

(c) Local maxima at $x = -3$ and $x = 5$, local minimum at $x = 1$

(d) Local maximum at $x = 5$, local minimum $x = 1$

(e) Local maximum at $x = 1$, local minima at $x = -3$ and $x = 5
8. A particle moves along the $x$-axis and its position at time $t$ is given by $x(t) = 400t - t^3$ for $0 \leq t$ where $t$ is measured in seconds and $x$ in feet. What is the average velocity from $t = 5$ to $t = 10$?

(a) 212.5 ft/sec
(b) 225 ft/sec
(c) 231.25 ft/sec
(d) 337.5 ft/sec
(e) 343.75 ft/sec

9. As in Problem #8, a particle moves along the $x$-axis and its position at time $t$ is given by $x(t) = 400t - t^3$ for $0 \leq t$ where $t$ is measured in seconds and $x$ in feet. What is the instantaneous velocity at $t = 7.5$?

(a) 212.5 ft/sec
(b) 225 ft/sec
(c) 231.25 ft/sec
(d) 337.5 ft/sec
(e) 343.75 ft/sec

10. Find the maximum area of a rectangle that is inside the triangle formed by the $x$-axis and the lines $y = -3x + 12$ and $y = 3x + 12$ if the base of the rectangle is on the $x$-axis and the two upper vertices are on the lines $y = -3x + 12$ and $y = 3x + 12$ as in the illustration.

(a) 30
(b) 24
(c) 18
(d) 12
(e) 9
11. The first graph on the left below is the graph of \( y = f(x) \). Which of the graphs labeled (a), (b), (c), (d) and (e) best represents the graph of \( y = -f(x + 1) \)?

![Graphs](image)

12. The second derivative of the function \( f(x) \) is \( f''(x) = 16x - x^3 \). Find the \( x \)-coordinate of each inflection point of the function \( f(x) \).

(a) Only inflection point is at \( x = 0 \)
(b) Only inflection point is at \( x = 4 \)
(c) There are two inflection points: at \( x = -4 \) and at \( x = 4 \)
(d) There are three inflection points: at \( x = -4 \), at \( x = 0 \) and at \( x = 4 \)
(e) There are no inflection points
Part III, Calculators Allowed

1. Answer the questions below based on the following information about the function \( f \). You must justify your answers.

   (i) The function \( f \) is continuous and differentiable for all values of \( x \).

   (ii) \( f(x) < 0 \) for \( x < 0 \); \( f(x) > 0 \) for \( 0 < x \).

   (iii) \( f''(x) < 0 \) for \( -6 < x < -2 \) and \( 5 < x \).

   (iv) \( f''(x) > 0 \) for \( x < -6 \) and \( -2 < x < 5 \).

   (v) \( f'''(x) < 0 \) for \( x < -4 \) and \( 3 < x < 7 \).

   (vi) \( f'''(x) > 0 \) for \( -4 < x < 3 \) and \( 7 < x \).

(a) On which intervals is the function decreasing?

(b) What is the \( x \)-coordinate of each local maximum (if any)?

(c) On which intervals is the function concave up?

(d) What is the \( x \)-coordinate of each inflection point (if any)?
2. Use the following table of values for (a), (b) and (c) below

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$g(x)$</th>
<th>$f'(x)$</th>
<th>$g'(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>4</td>
<td>-2</td>
<td>-3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>1</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2</td>
<td>13</td>
<td>-4</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>3</td>
<td>10</td>
<td>-8</td>
</tr>
</tbody>
</table>

(a) Find $b'(4)$ for $b(x) = \frac{g(x)}{f(x)}$.

(b) Find $h'(3)$ for $h(x) = g(f(x))$.

(c) Find $k'(2)$ for $k(x) = (f(x))^3$. 
3. Find the absolute maximum and absolute minimum values of the function \( f(x) = 2x^3 - 150x^2 + 50,000 \) on each interval.

(a) \(-20 \leq x \leq 20\)

(b) \(-10 \leq x \leq 60\)
4. A large rectangular area is to be fenced off as in the diagram below (a large rectangle divided into two smaller rectangles). The fence used to divide the space costs $10 per foot and the fence used for the perimeter costs $15 per foot. If the total budget for the project is $60000, what are the dimensions which yield the largest area?
5. A spotlight at ground level is located 40 feet from a very tall building, directly in front of the door into the building. A 6 feet tall woman exits the building and walks directly towards the light. If she is walking at 5 feet per second, how fast is the length of her shadow on the building changing when she is 10 feet from the building?