STAT 3128 EXAM Solutions Fall 2008 Instr. Sonin

NAME__________

(100 + 5) **Show all work on problems ! You can use formula sheet and the tables !**

(8) 1. Consider the following data set

-1 9 3 13 11 1 5 17

Find each of the following: a) mean, b) median, c) range, d) sample variance,

e) sample st. deviation, f) z-score for $x = -1$.

a) mean $= \frac{58}{8} =$ **7.25**, b) median $= \frac{5+9}{2} =$ **7**, c) range **18**, d) sample var-ce $s^2 = \mathbf{39.357}$,

e) sample st. dev. $s = \mathbf{6.274}$, f) z-score for $x = -1$; $= \frac{-1-7.25}{6.274} = \mathbf{-1.315}$

(8) 2. The probability that an automobile battery has a defective terminal is .07, the probability that it has a defective plate is .05, and the probability that it has both a defective terminal and a defective plate is .03. What is the probability that

a) a battery has at least one defect ? **Sol-n:** Let $A = \{$defective terminal$\}$, $B =$ defective plate$\}$

$P($ at least one defect $) =$ $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .07 + .05 - .03 = \mathbf{.09}$,

b) a battery has at most one defect ? $P(...) = 1 - P($both def$) = 1 - P(A \cap B) = 1 - .03 = \mathbf{.97}$

c) a battery has neither of these defects? $P(...) = 1 - P(A \cup B) = 1 - .09 = \mathbf{.91}$.

(9) 3. A laboratory will test six items denoted by A, B, C, D, E and F. The order in which these items are tested is important.

a) In how many orders may the items be tested ? $6! = \mathbf{720}$

Assuming that the orders are equally likely,

b) what is the probability that item A will be tested among the first four ?

$$P(...) = \frac{\binom{4}{1}5!}{6!} = \frac{2}{3} \approx \mathbf{.667}$$

c) what is the probability that items A and B will be tested among the first four ?

$$P(...) = \frac{\binom{4}{1}\binom{3}{1}4!}{6!} = \mathbf{\frac{2}{5} = .4}$$

(8) 4. A company uses three different assembly lines $- A_1,\ A_2$ and A_3 to produce a particular component. Suppose that 2 % of components produced by line A_1, are defective, that 1 % of components produced by line A_2 are defective, and that 3 % of components produced by line A_3 are defective. Suppose that 50 % of all components produced by line A_1, 20 % produced by line A_2, and 30% come from line A_3.

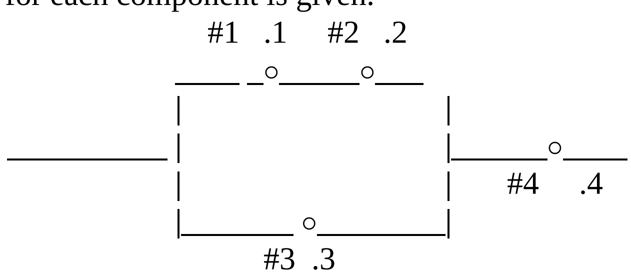
a) What is the probability that a randomly selected component is defective ?

$$P(D) = \sum P(A_i)(P(D \mid A_i) = (.5)(.02) + (.2)(.01) + (.3)(.03) = \mathbf{.021}$$

b) If a randomly selected component is defective, what is the probability that it came from line A_1 or A_2 ?

$$P(A_1 \cup A_2 | D) = P(A_1 | D) + P(A_2 | D) = \frac{P(A_1)P(D \mid A_1) + P(A_2)P(D \mid A_2)}{\sum P(A_i)(P(D \mid A_i)} = \frac{(.5)(.02)+(.2)(.01)}{.021} \approx \mathbf{.571}$$

(10) 5. A system comprised of four independent components. The probability of *failure* for each component is given.



a) Find the probability that the system operates properly.

$$A = A_4 \cap [(A_1 \cap A_2) \cup A_3]\,;\ P(A) = P(A_4)[P(A_1)P(A_2) + P(A_3) -$$

$$- P(A_1)P(A_2)P(A_3)] = (.6)[(.9)(.8) + (.7) - (.9)(.8)(.7)] = \mathbf{.5496}.$$

b) Given that the system operates properly, what is the probability that the component # 2 operates properly.

$$P(A_2 | A) = \frac{P(A_2 \cap A)}{P(A)} = \frac{P(A_4)[P[(A_2 \cap (A_1 \cup A_3)]}{P(A_4)P[(A_1 \cap A_2) \cup A_3]} = \frac{(.6)(.776)}{(.6)(.916)} \approx \mathbf{.847}.$$

(9) 6. Light bulbs produced by a manufacturer are known to last 600 hours or more with probability .9. A carton of 20 bulbs is purchased.

X = # of bulbs that last... is a Bin ($n = 20,\ p = .9$)

a) What is the probability that at least 15 of them last at least 600 hours ?

$$P(X \geq 15) = 1 - P(X \leq 14) = 1 - .011 = \mathbf{.989} \text{ (using Table) ;}$$

b) What is the probability that all of the bulbs last at least 600 hours ?

$(.9)^{20} \approx \mathbf{.122}$

c) What is the expected value and variance of the number of bulbs that last at least 600 hours ? $EX = np = 20(.9) = \mathbf{18}$, $V(X) = npq = 20(.9)(.1) = \mathbf{1.8}$

(7) 7. Suppose X is a rv with probability function

x	-2	-1	0	1	5
$p(x)$	.1	.2	.2	.4	.1

a) Find P($X \leq -2$), P($X \leq 1$), P($0 \leq X \leq 3$)

.1 **.9** **.6**

b) Calculate the expected value of X, the variance, the standard deviation.

$EX = -.2 - .2 + .4 + .5 = \mathbf{.5}$

$EX^2 = .4 + .2 + .4 + 2.5 = 3.5$

$V(X) = EX^2 - (EX)^2 = 3.5 - (.5)^2 = \mathbf{3.25}$, **st.dev** $\approx \mathbf{1.8}$

c) Calculate the expected value of $2 + \sqrt{X+2}$. $E[2 + \sqrt{X+2} =$

$= 2 + \sum_x \sqrt{x+2}\, p(x) = 2 + (.2)\sqrt{1} + (.2)\sqrt{2} + (.4)\sqrt{3} + (.1)\sqrt{7} \approx \mathbf{3.44},$

(7) 8. Let X denote the demand for . . . and suppose that the pdf of X is

$$f(x) = \begin{cases} cx^2 & \text{for } 1 \leq x \leq 3, \\ 0 & \text{otherwise.} \end{cases}$$

Find $P(-1 \leq X \leq 2)$, $P(2 \leq X \leq 3)$, the expected value of X.

Sol: $F(x) = 0$, if $x \leq 1$, $F(x) = 1$, if $x \geq 3$, $F(x) = c\int_1^x y^2 dy = c\frac{y^3}{3}|_1^x = \frac{\boldsymbol{c(x^3-1)}}{\mathbf{3}}$, $1 \leq x \leq 3$,

$F(3) = 1 = c\frac{26}{3}$ and hence $\boldsymbol{c = \frac{3}{26}} \approx .115$;

$P(-1 \leq X \leq 2) = F(2) - F(-1) = \frac{c7}{3} - 0 = \frac{7}{26} \approx \mathbf{.269}$;

$P(2 \leq X \leq 3) = F(3) - F(2) = 1 - \frac{7}{26} \approx \mathbf{.731}$;

$\boldsymbol{EX =} \int x f(x) dx = c\int_1^3 x^3 dx = c\frac{1}{4}x^4\,|_1^3 = \frac{c(81-1)}{4} = \frac{60}{26} \approx \mathbf{2.31}$

(9) 9. Let X and Y be discrete random variables with joint probability function (pmf) given by the following table

$x\backslash y$	0	1	3	5	$p_X(x)$
1	0	.15	.03	.02	**.2**
2	.1	.05	.25	.08	**.48**
3	.05	.2	.07	0	**.32**
$p_Y(y)$	**.15**	**.4**	**.35**	**.1**	

a) What is the probability $P(X + Y > 2)$?

$$P(X + Y > 2) = 1 - P(X+Y \leq 2) = 1 - [p(1,0) + p(1,1) + p(2,0)] =$$

$$= 1 - [0 + .15 + .1 = \mathbf{.75} \text{ ;}$$

b) Find the expected value of Y. $EY = 1(.4) + 3(.35) + 5(.1) = \mathbf{1.95}$

c) Are X and Y independent ? **No,** e.g. $p(1,0) \neq p_X(1)p_Y(0)$

(10) 10. A weight-loss company claims the average weight-loss of dieters using its prepackaged meals is more than 5 lbc. The Food and Drug Administration studies 55 dieters, and finds that they have a mean weight loss of 5.4 lbs., with standard deviation 2.4 lbs.

a) Set up the null and alternative hypotheses to test the company's claim.

$$\boldsymbol{H_0 : \mu = 5 \ ; \ H_a : \mu > 5}$$

(b) Find the value of the test statistic. $z = \frac{5.4-5}{2.4/\sqrt{55}} \approx \mathbf{1.24}$

(c) Find the observed significance level (p-value) of the test.

$$1 - \Phi(1.24) = 1 - .8925 = \mathbf{.1075}$$

d) What would you conclude at the $\alpha = .1$ level of significance ?

For $\alpha = .1$ rej. region starts at z $= 1.282 > 1.24$. Hence **fail to reject;**

or ***p*-value** $=$ **.1075 > .1** and hence fail to reject.

(9) 11. The breakdown voltage of a randomly chosen diode of a certain type is known to be normally distributed with mean value 40 V and standard deviation 1.5 V.

a) What is the probability that the voltage of a single diode is between 39 and 42 ?

$$P(39 < X < 42) = P(\tfrac{39-40}{1.5} \approx -.67 < Z < \tfrac{42-40}{1.5} \approx 1.33) =$$

$$= .9082 - .2514 = \mathbf{.6568}$$

b) What value is such that only 15% of all diodes have voltage exceeding that value ?

$.15 \approx P(Z > 1.04) = P(X > \frac{c-40}{1.5})$, hence $c = 40 + 1.5(1.04) = \mathbf{41.56}$.

c) If four diodes are independently selected, what is the probability that at least one has a voltage exceeding 42 ?

$$P(....) = 1 - P(\text{none....}) = 1 - (.9082)^4 \approx \mathbf{.32}$$

(6) 12. In a survey of 500 vacationers from USA, 240 said that they are using on-line service to make a reservation for lodging.

a) Construct the 90% confidence interval for the proportion of vacationers who use an on-line service.

Large sample, $\alpha = .1, \quad z_{\alpha/2} = z_{.05} = 1.645$

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\,\hat{q}}{n}} = \frac{240}{500} \pm 1.645\sqrt{\frac{(.48)(.52)}{500}} = .48 \pm .037 = (\mathbf{.443}, \mathbf{.517}).$$

b) What sample size is necessary if the 90% confidence interval is to have width of at most .05 ?

$$B = \frac{w}{2} = .025, \ n = (\frac{1.645\sqrt{(.5)(.5)}}{.025})^2 \approx \mathbf{1083}$$

(+ 5) * 13. A $2 \times 2 \times 2$ wooden cube is painted on 5 faces and then cut into 8 unit cubes. One unit cube is randomly selected and rolled. What is the probability that exactly two of the five visible faces are painted ?

Sol-n: Such cube consists of 4 small cubes with three sides painted, and 4 two sides painted.

$P(\text{to see exactly two p-d sides}) = P(\text{two-sided selected}) * P(\text{it falls on } \textit{unpainted} \text{ side}) +$

$+ P(\text{three-sided selected}) * P(\text{it falls on } \textit{painted} \text{ side}) = \frac{4}{8} * \frac{4}{6} + \frac{4}{8}\frac{3}{6} = \mathbf{\frac{7}{12} \approx .58}$