

Unsquare Party, 2098

Ashley noticed that the set of ages of her relatives, all of whom were whole numbers in the range $1, \dots, 100$ has the unusual property that no two of them multiplied together is a perfect square? **What is the largest number of relatives Ashley could have?**

Solution: The answer is 62. Ashley could have only 1 relative with an age in the set $\{1, 4, 9, 16, 25, \dots, 100\}$ of perfect squares. Likewise, she could have at most one in the set $\{2, 8, 18, 32, 50, 72, 98\}$. Continuing in this fashion, there are 62 different sets of numbers such that the product of any two members of the set is a perfect square. A 62-member set could be obtained by picking one number from each set. Note to Editor, the relation $x \approx y$ if and only if $x \cdot y$ is a perfect square is an equivalence relation, so, for example the sets $[1] = \{1, 4, 9, 16, 25, \dots, 81\}$ and $[2] = \{2, 8, 18, 32, 50, 72, 98\}$ are disjoint.