

Multiple Quotients, 1696

Brackets can be inserted into the expression $1 \div 2 \div 3 \div 4$ in various ways. For example, $(1 \div 2) \div (3 \div 4) = 2/3$, whereas $1 \div ((2 \div 3) \div 4) = 6$. Similarly, brackets can be inserted into $1 \div 2 \div 3 \div 4 \div 5 \div 6 \div 7 \div 8 \div 9 \div 10 \div 11$ to produce a large collection of whole numbers. **What do you get if you divide the largest of these whole numbers by the smallest of these whole numbers?**

Solution: The numbers are precisely $(1abc\dots)/(2uvw\dots)$ where each of the numbers $1, 2, 3, 4, \dots, 11$ appears once. Another way to write these numbers is as $11!/(2uvw\dots)^2$ where u, v, w, \dots are among $3, 4, \dots, 11$. The largest integer is $11!/2 = 9979200$. Since $11! = 11x7x5x5x3x3x3x2x2x2x2x2x2x2 = 11x7x(2x5x8x9)^2$, it follows that the smallest integer is $11!/(2x5x8x9)^2 = 77$. The quotient in question is $9979200/77 = 129600$.