1. The numbers $1, 2, 3, \ldots, 100$ are arranged in a 10×10 grid so that consecutive numbers occupy adjacent squares. What is the greatest possible sum of the numbers along the diagonal of the grid.

   Solution: 870. For convenience we first solve the minimum problem. To minimize the sum of the numbers on the (main) diagonal, we should start on the diagonal and stay include as many of the first few numbers as possible on the diagonal. If we start in the upper left corner, note that only odd numbers will appear on the diagonal. We can zigzag our way down the diagonal, putting the numbers $1, 3, 5, 7, 9, 11, 13, 15$, and 17 on the diagonal as shown. At this stage we cannot hope to put 19 on the diagonal because we would then not have access to both the squares above the diagonal and below the diagonal. We can, however use up all the squares above the diagonal, then use the final diagonal square, and finally complete the arrangement by listing all the rest of the number in the square below the diagonal. This results in putting the number 59 in the bottom right corner, so the sum of the entries on the diagonal is $1 + 3 + \cdots + 17 + 59 = 140$. In general, for a $2n \times 2n$ grid, the smallest sum on numbers along the diagonal is the sum of the first $2n - 1$ positive odd integers plus the number $2n^2 + 2n - 1$. Hence the smallest possible sum is $(2n - 1)^2 + 2n^2 + 2n - 1 = 6n^2 - 2n$. To get the maximum possible sum, we can replace each diagonal number k by $101 - k$. The result is $8n^3 - 6n^2 + 4n$, which for $n = 5$ gives the value 870.