Face Painting, 2094

Suppose some faces of a large wooden cube are painted red and the rest are painted black. The cube is then cut into unit cubes. The number of unit cubes with some red paint is found to be exactly 200 larger than the number of cubes with some black paint. **How many cubes have no paint at all?**

Solution: The cube is $11 \times 11 \times 11$ and there are two adjacent black faces. Thus $4n^2 - 5n + 2 - (2n^2 - n) = 200$, from which it follows that $n = 11$. Another approach is to count the number of red only, black only and red and black cubes. The table below shows these figures for small values of n.

n	Black only	Red only	Red and Black	$n^3 - (n-2)^3$
1	0	0	1	1
2	0	2	6	8
3	3	11	12	26
4	10	28	18	56
5	21	53	24	98
n	$2n^2 - 7n + 6$	$4n^2 - 11n + 8$	$6n - 6$	$6n^2 - 12n + 8$

Thus we can solve the equation $4n^2 - 11n + 8 - (2n^2 - 7n + 6) = 200$ to get $n = 11$. **Hence there are** $(11-2)^3 = 729$ **unpainted cubes.**