

Bug Box

A bug starts at a point on a large plane surface. On day one he walks one metre north and on day two he carries on from there and walks two metres east. At the end of day two, the bug is the closest to the starting point as he will be for over a week.

On days 3 through 16 the bug walks the number of meters equal to the day number and in a direction which is at right angles to the previous day's route. The bug continues this until after 16 days, it finds itself back at the origin. Its path does not intersect itself. **In order, which direction did he walk on each day.**

Solution: Note that the vertical sides of the polygon are all odd lengths. Since the up sides equals the down sides, we must partition the set $\{1, 3, 5, 7, 9, 11, 13, 15\}$. If he walks south on day three, he would get closer to the origin, a contradiction. So one set must have both 1 and 3 the sum of the members of each subset is $(1 + 3 + 5 + 7 + 9 + 11 + 13 + 15) \div 2 = 32$. Experimentation shows that this can be done in only one way: $N = \{1, 3, 13, 15\}$ and $S = \{5, 7, 9, 11\}$.

The horizontal edges are trickier. The sum $2 + 4 + 6 + \dots + 16 = 72$ so the partition must be into two sets each with sum 36 such that the 'right' edges include both 2 and 4. The reason for this is that if 2 and 4 are not in the same subset, then the bug gets closer to the starting point during the fourth day, a contradiction. There are four ways to partition the even numbers into two subsets so that 2 and 4 are in the same subset. Case I: $\{2, 4, 14, 16 | 6, 8, 10, 12\}$, Case II: $\{2, 4, 6, 10, 14 | 8, 12, 16\}$, Case III: $\{2, 4, 6, 8, 16 | 10, 12, 14\}$, Case IV: $\{2, 4, 8, 10, 12 | 6, 14, 16\}$. Only two of these give rise to non-intersecting paths. The path for $E = \{2, 4, 14, 16\}$, $W = \{6, 8, 10, 12\}$ has the property that the bug gets closer to home during day 6. So we have partition $E = \{2, 4, 6, 8, 16\}$, $W = \{10, 12, 14\}$, $N = \{1, 3, 13, 15\}$ and $S = \{5, 7, 9, 11\}$, which gives rise to the trail **NENESESESWSWN-WNE**

