Back to Front, 1785

Given a stack of 11 cards numbered $11, 10, 9. \ldots, 1$, we wish to reverse their order to give $1, 2, 3, \ldots, 11$. To do this we are allowed at any stage to make a 'move' of the following type: remove any section of adjacent cards from the pack and insert them elsewhere in the pack. For example, one move that you could start with is to reposition $9, 8, 7$ to give the ordering $11, 10, 6, 5, 9, 8, 7, 4, 3, 2, 1$. **What is the minimum number of moves required to reverse the 11 cards?**

Solution: First we describe a six-move sequence that works. Then we prove that no shorter sequence will do.
$11, 10, 9, 8, 7, 6, 5, 4, 3, (2, 1)$;
$(11, 10, 9, 8, 2), 1, 7, 6, 5, 4, 3$;
$1, 7, 6, 5, (4, 11), 10, 9, 8, 2, 3$;
$1, 7, 6, (5, 10), 9, 8, 2, 3, 4, 11$;
$1, 7, (6, 9), 8, 2, 3, 4, 5, 10, 11$;
$1, (7, 8), 2, 3, 4, 5, 6, 9, 10, 11$;
$1, , 2, 3, 4, 5, 6, 7, 8, 9, 10, 11$
On the other hand, six moves is the best possible. To see this, call two consecutive cards a, b a disorder if $a > b$. Hence, we have 10 disorders initially. The first and last moves can remove at most one disorder, and any other move can diminish the number of disorders by at most two (if three are removed, then one is created). **So the fewest number of moves needed to remove all the disorders is** $1 + 1 + 8/2 = 6$.