Math 1165 **Discrete Math** **Test 2**

April 6, 1995 **Show all work** **Name** ______________________________

1. Let U be the set of integers from 1 to 1000 (i.e., $U = \{1, 2, 3, \ldots, 1000\}$).

   (a) Prove that every 501 element subset S of U has a pair of elements which differ by exactly 1.

   (b) Prove that every 501 element subset S of U has a pair of elements which differ by exactly 10.

   (c) Prove that every 501 element subset S of U has a pair of elements which differ by exactly 100.

   (d) Must every 501 element subset of U have a pair of elements which differ by 3? Explain in detail.

2. (a) Find the greatest common divisor d of 381 and 1995.

   (b) Find the least common multiple of 381 and 1995.

   (c) Use the Euclidean algorithm to find integers m and n satisfying

   $$d = 1995m + 381n,$$

   where d is the gcd of 1995 and 381.

3. Notice that

$$\begin{aligned} 1 &= 1 && (1)\\ 1+3 &= 4 && (2)\\ 1+3+5 &= 9 && (3)\\ 1+3+5+7 &= 16 && (4) \end{aligned}$$

(a) List the next three equations suggested by the pattern.

(b) Given that the four equations above are the $1^{st}, 2^{nd}, 3^{rd}$, and 4^{th}, write the n^{th} equation of the sequence.

(c) Use mathematical induction to prove that the n^{th} equation is true for all positive integer values of n.

4. Find the base five representation of 255 in two ways.

   (a) By repeated division.

   (b) By repeatedly subtracting powers of five.

5. Find the Boolean product shown below:

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \odot \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix}$$

6. For each of the four sets of conditions listed below, describe the (some) smallest subset of the real numbers satisfying the conditions. For example if the conditions were (a) $1 \in S$ and (b) $\forall x, \quad x \in S \rightarrow x+1 \in S$, your answer would be: the set of positive integers.

   A. (a) $0 \in A$
      (b) $\forall x, \quad x \in A \rightarrow (x+2 \in A \quad \bigwedge \quad x-2 \in A)$

   B. (a) $0 \in B$
      (b) $\forall x, \quad x \in B \rightarrow (x+2 \in B \quad \bigvee \quad x-2 \in B)$

   C. (a) $3 \in C \ \bigwedge \ 8 \in C$
      (b) $\forall x \forall y, \quad (x \in C \quad \bigwedge \quad y \in C) \rightarrow (x-y \in C)$

   D. (a) $0 \in D \ \bigwedge \ 1 \in D$
      (b) $\forall x, \quad x \in D \rightarrow x+2 \in D$