

September 25, 2000

Your name _____

1. (10 points) Find the base 6 representation of 2000.
2. (10 points) Find the base 6 representation of $1/5$.
3. (10 points) Find a pair of relatively prime integers m and n for which $\frac{m}{n} = 1.2\bar{3}$.
Two numbers are relatively prime if their greatest common divisor is 1.
4. (10 points) Find a base 7 digit d such that $2d16_8 = d405_7$.

5. (10 points) Find the best (winning) move in the game of Bouton's Nim (17, 13, 12, 11).

6. (12 points) Let $M = 161,161$ and let $N = 12,376$.

(a) Compute $LCM(M, N)$

(b) Compute $GCD(M, N)$

(c) Find the number of divisors of M .

7. (20 points) Look at the four equations below.

$$\begin{aligned}2 &= 2 \cdot 1 \\2 + 4 &= 3 \cdot 2 \\2 + 4 + 6 &= 4 \cdot 3 \\2 + 4 + 6 + 8 &= 5 \cdot 4\end{aligned}$$

a. Write the next three equations in the sequence.

b. If the four equations above correspond to $k = 1, 2, 3,$ and $4,$ what is the n th equation?

c. Prove by mathematical induction that the n th equation is true for all integers $n \geq 1.$

8. (10 points) Find the representations of the integers 1 through 13 in base $-6.$

9. (15 points) Solve the equation $123x + 456y = 3$ for integers x and y .

10. (13 points) Prove that $2^n \leq n!$ for all integers $n \geq 4$.