Math 1165 **Discrete Math** **H. Reiter** **Test 1**

Feb 23, 1995 **Show all work.** **Name** _____

1. A certain computer company employs 100 computer programmers. Of these, 29 can program in FORTRAN *only*, 22 can program in Pascal *only*, 18 can program in COBOL *only*, 8 can program in FORTRAN and Pascal, 9 can program in FORTRAN and COBOL, 7 can program in COBOL and Pascal, and 11 can program in none of the three languages.

   (a) How many programmers can program in all 3 languages?

   (b) How many programmers can program in exactly two of the three languages?

2. Classify each statement below as a tautology, a contradiction, or a contingency. You do not need to prove your answers.

   (a) $(p \to q) \leftrightarrow (\neg p \bigvee q)$

   (b) $(p \to q) \leftrightarrow (p \bigwedge \neg q)$

   (c) $(p \bigwedge (q \to r)) \leftrightarrow ((p \bigwedge q) \to (p \bigwedge r))$

3. Translate into the language of propositions and classify each of the following as true or false. You do not need to prove your answers.

   (a) Implication is associatve.

   (b) Disjunction (or) distributes over implication.

   (c) Biconditional distributes over implication.

   (d) Implication is commutative.

4. Suppose $A(x,y), B(x,y), C(x,y)$ and $D(x,y)$ are predicates defined for all real numbers x and y and A and B are negations of each other. Suppose also that C and D are negations of each other. For example, if $A(x,y)$ meant $x = y$ then $B(x,y)$ would mean $x \neq y$. Write the negation of the predicate expressions below without using the $\neg$ symbol. For example, $\forall x A(x,x)$ is negated by $\exists x B(x,x)$.

   (a) $\forall y \exists x \ A(x,x) \bigvee C(x,y)$

   (b) $\forall x \exists y \ A(x,y) \rightarrow C(x,y)$

   (c) $\forall x \forall y \ A(y,y) \leftrightarrow C(x,x)$

5. Describe each of the following statements as **A** for **always** true, **S** for **sometimes** true, and **N** for **never** true. Explain carefully why your answer is correct. If you say the statement is sometimes true, you must provide a case where it is true and one where it is false. For example, if we say a and b are numbers and $a+b=1$, your answer should be sometimes (S), and you should give an example of a choice of a and b for which $a+b=1$ and one for which $a+b\neq 1$. In what follows below, $A, B,$ and C are set variables.

   (a) $(A \bigcap B) \bigcup (A \bigcap C) = A \bigcap (B \bigcup C)$

   (b) $A \times (B \bigcap C) = (A \times B) \bigcap (A \times C)$

   (c) $(A \bigcap B) \times (A \bigcap C) = (A \bigcap (B \times C))$

   (d) The set A has exactly 9 subsets.

   (e) $|(A \bigcup P(A))| = 10$ (Hint: Construct A so that A and $P(A)$ overlap.)