March 21, 1995 **Show all work.** **Name** ____________________

1. The sets A, B, C and D are defined below:

   $A = \{1, 2, 3\}$,
   $B = \{3, 4, 5\}$,
   $C = \{1, 5, 6\}$, and
   $D = \{1, 2, \{1\}, \{2\}\}$.

   In each part below, find the set itself or the number of elements of the set, whichever is requested.

   (a) $|(A \bigcup B \bigcup C) \times (A \bigcup B \bigcup C)|$

   (b) $(A \bigcap B) \bigcup (C \bigcap B) \bigcup (A \bigcap C)$

   (c) $(D \bigcap P(D))$

   (d) $|P(A \bigcup B \bigcup C)|$

   (e) $|P(D \times D)|$

   (f) $|P(D) \times P(D)|$

   (g) $|(A \times A) \bigcup (B \times B) \bigcup (C \times C)|$

   (h) $|P(D) \bigcap P(A)|$

2. Let $U = \{1, 2, 3, \ldots, 1000\}$ and let A_2, A_3, and A_5 denote the subsets of U defined as follows:
$A_2 = \{n | 1 \leq n \leq 1000 \text{ and } n \text{ is even }\}$,
$A_3 = \{n | 1 \leq n \leq 1000 \text{ and } n \text{ is a multiple of } 3\}$,
$A_5 = \{n | 1 \leq n \leq 1000 \text{ and } n \text{ is a multiple of } 5\}$,
All complements are taken with respect to U. Find the number of elements of each of the sets listed below.

    (a) $A_2 \bigcap A_3 \bigcap A_5$

    (b) $A_2 \bigcap A_3 \bigcap \overline{A_5}$

    (c) $A_2 \bigcap \overline{A_3} \bigcap A_5$

    (d) $\overline{A_2} \bigcap A_3 \bigcap A_5$

    (e) $A_2 \bigcap \overline{A_3} \bigcap \overline{A_5}$

    (f) $\overline{A_2} \bigcap A_3 \bigcap \overline{A_5}$

    (g) $\overline{A_2} \bigcap \overline{A_3} \bigcap A_5$

    (h) $\overline{A_2} \bigcap \overline{A_3} \bigcap \overline{A_5}$

3. Classify each statement below as a tautology, a contradiction, or a contingency. You do not need to prove your answers.

    (a) $[(r \rightarrow s) \bigwedge (q \rightarrow r) \bigwedge (p \rightarrow q) \bigwedge p] \rightarrow s$

    (b) $(p \bigvee q) \bigwedge (r \bigvee s) \bigwedge (u \bigvee v) \leftrightarrow [(p \bigvee r \bigvee u) \bigwedge (q \bigvee s \bigvee v)]$

4. Let P, Q, S, SR, N, and $=$ be predicates on the real numbers as defined below:
$I(x)$: x is an integer,
$Q(x)$: x is a rational number,
$P(x)$: x is a positive number,
$N(x)$: x is a negative number,
$S(x)$: x is the square of an integer,
$SR(x)$: x is the square root of an integer,
$=(x, y)$: $x = y$. (we'll use the standard 'infix' notation)
Construct predicate expressions using $\bigwedge, \bigvee, \neg, \rightarrow, \leftrightarrow$, constants, and the predicates above with the meanings given below:

   (a) The number 6 is a negative rational number if and only if π^2 is an integer.

   (b) If the square root of an integer is a rational number, then it is an integer.

   (c) Some rational numbers are positive.

   (d) Any number which is neither positive nor negative is an integer.