

3. Let $S = \{1, 2, 3, 4, 5, 6, 7\}$.

- (a) How many non-empty subsets does S have?
- (b) How many subsets of S consist entirely of odd numbers?
- (c) How many subsets of S have exactly 4 elements?
- (d) How many four element subsets of S contain exactly two odd numbers?
- (e) How many four digit numbers can be made using the digits of S if a digit may be used repeatedly?
- (f) How many four digit numbers can be made using the digits of S if a digit may be used only once?
- (g) How many even four digit numbers bigger than 2000 can be made using the digits of S if a digit may be used only once?
- (h) How many four digit numbers can be made using the digits of S if two consecutive digits must differ by more than 1? For example, 5461 is not allowed because $5 - 4 = 1$, but 5363 is allowed.

4. Suppose $A, B,$ and C are sets of integers such that $B \cap C = \phi, |B| = 15, |C| = 12, |A \cap B| = |A \cap C| = 2,$ and $|A \cup B| = 28.$ Find each of the following:

(a) $|A \cap \overline{C}|$

(b) $|(A \cup B) \cap \overline{C}|$

(c) $|A \cup B \cup C|$

5. Look at the four equations below.

$$2 = 2 \cdot 1$$

$$2 + 4 = 3 \cdot 2$$

$$2 + 4 + 6 = 4 \cdot 3$$

$$2 + 4 + 6 + 8 = 5 \cdot 4$$

- (a) Write the next three equations in the sequence.
- (b) If the four equations above correspond to $n = 1, 2, 3,$ and $4,$ what is the n th equation?
- (c) Prove by mathematical induction that the n^{th} equation is true for all integers $n \geq 1.$

6. Let $A = \{1, 2, 3\}$.

(a) Give an example of a 3×3 boolean matrix which represents a relation on A that is both symmetric and antisymmetric. Which entries of the matrix can be either 0 or 1? Use this information to count the number of relations on A which are both symmetric and antisymmetric. (Remember that there are 2^9 relations on A).

(b) How many relations on A are antisymmetric?

(c) How many relations on A are symmetric?

(d) How many relations on A are reflexive, symmetric, and transitive?

7. Let $A = \{1, 2, 3, 4\}$. Find examples of relations on A which satisfy each of the following collections of conditions:

- (a) Reflexive and symmetric and not transitive.

- (b) Reflexive and antisymmetric and not transitive.

- (c) Symmetric and transitive and not antisymmetric.

- (d) Symmetric and not transitive and not reflexive.

8. Consider the fragment of Pascal code below:

```
if N <= 3 then N := 2 * N - 1 else N := N - 4;  
if N <= 5 then N := N - 5 else N := 3 * N
```

Suppose N is an integer variable. Find a function f which describes the effect of the entire fragment on variable N .

9. Again assume N is an integer variable. What is the effect of the fragment below on N ? Express your answer as a function. Explain your answer.

```
If  $N \leq 0$  then  $N := 100$ ;  
While  $N > 0$  do  
  Begin  
     $N := N - 4$   
  End
```