

1. Let  $\mathcal{N}$  denote the three-digit integers; that is the set of integers from 100 through 999.
  - (a) What is the cardinality of  $\mathcal{N}$ ?
  - (b) How many members of  $\mathcal{N}$  contain exactly one even digit?
  - (c) How many odd integers are there in  $\mathcal{N}$ ?
2. A bank password consists of two letters of the English alphabet (26 letters) followed by four digits. How many different passwords are there? How many different passwords have distinct symbols? (Letter “o” is not the digit zero!)
3. How many arrangements of no more than three letters can be formed using the letters of the word SUCCESS. A letter can be used only as many times as it appears in the word.
  - (a) with repetitions allowed?
  - (b) without repetitions?

4. Prove that

$$\binom{n}{r} + 2\binom{n}{r+1} + \binom{n}{r+2} = \binom{n+2}{r+2} \text{ for } 2 \leq r+2 \leq n$$

Hint: Use the Pascal identity:  $\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$ .

5. A fair coin is tossed three times, and  $H$  or  $T$  is recorded after each toss.
  - (a) Construct the sample space.
  - (b) Construct the tree diagram representation of the sample space.
  - (c) Let  $A = \{(H, x, y) | x, y \in \{H, T\}\}$ ,  $B = \{(x, H, y) | x, y \in \{H, T\}\}$ , and  $C = \{(x, y, H) | x, y \in \{H, T\}\}$ 
    - i. Find the probability of  $A$ .
    - ii. Find the probability of  $AB$
    - iii. Find the probability of  $ABC$
    - iv. Find the event that the three rolls result in exactly 2 H's in terms of  $A, B$ , and  $C$ .
6. Find ( for  $n = 2, 3, \dots, 10, 11$ ) the probability that a random  $n$ -digit decimal integer has distinct digits. This is 10 different questions.

7. Two points are randomly and simultaneously selected from the  $4 \times 5$  grid of 20 lattice points  $\{(m, n) | 1 \leq m \leq 5 \text{ and } 1 \leq n \leq 4\}$ . What is the probability that the distance between them is rational?
8. Four cards are randomly selected from the 52 card deck. Find the probability that a sample contains:
- (a) Exactly two spades.
  - (b) Exactly two spades and two red cards.
  - (c) Cards from four different suits.
  - (d) Two pairs of cards  $\{a, b\}, \{c, d\}$  where  $a$  and  $b$  are the same denomination and  $c$  and  $d$  are the same denomination, but different from  $a$  and  $b$ .
  - (e) Three or more cards of the same suit.
9. Five cards are drawn from a standard 52-card deck. What are the probabilities of the following poker combinations:
- (a) One pair (any two cards of the same denomination), but not anything *better*. You have to know something about poker to do this one. If you need to ask, please do so.
  - (b) Two pairs (any two pairs of cards of the same denomination plus a fifth card that has a different denomination).
  - (c) A straight (any five cards of more than one suit in sequence, like J H, 10 D, 9 S, 8 H, 7 D). You are expected to include the straight flushes in the count.
10. A box contains 2 pennies, 4 nickels, and 6 dimes. Six coins are drawn without replacement, with each of the 12 coins having the same probability of being chosen. What is the probability that the value of the coins is at least 50 cents?

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