Homework 4 Math 1165

Profs. S. Molchanov and H. Reiter

1. Prove that geometrical progression is increasing faster than perfect squares. Specifically: prove that for appropriate $n_0 > 0$ and any $n \geq n_0$

$$2^n > n^2.$$

2. Prove that for suitable $n \geq n_0$

$$n! \geq 5 \cdot 3^n.$$

   Find the minimal n_0.

3. Prove that

$$n > 5 \log_3 n$$

   if $n \geq n_0$. Find the minimal n_0.

4. Prove that for any arithemetic progression with the first term a and difference d then

   (a) $a + (a + d) + \cdots + a + (n - 1)d = n \cdot a + \frac{d(n-1)n}{2}$ and

   (b) $a + (a + d) + \cdots + a + (n - 1)d = \frac{n}{2}(a + a + (n - 1)d)$ (the sum is the average of the first and last terms).

5. For each of the sets of conditions listed below, describe the smallest subset of the real numbers satisfying the conditions. If there is more than one set that satisfies the conditions, state that and say why. For example, if the conditions were (a) $1 \in S$ and (b) $\forall x, \quad x \in S \rightarrow x + 1 \in S$, your answer would be: the set of positive integers. The reason for your answer could be that this is what Mathematical Induction guarantees. Recall that $\bigwedge$ means 'and', $\forall$ means 'for all', $\exists$ means 'there exists'

   A. i. $0 \in A$
      ii. $\forall x, \quad x \in A \rightarrow (x + 2 \in A \quad \bigwedge \quad x - 2 \in A)$

   B. i. $0 \in B$
      ii. $\forall x, \quad x \in B \rightarrow x - 1 \in B$

   C. i. $2 \in C \bigwedge 4 \in C$
      ii. $\forall x \forall y, \quad (x \in C \quad \bigwedge \quad y \in C) \rightarrow (x - y \in C)$

   D. i. $0 \in D \bigwedge 1 \in D$
      ii. $\forall x, \quad x \in D \rightarrow x + 2 \in D$

   E. i. $0 \in E \bigwedge 1 \in E$
      ii. $\forall x, \quad x \in E \rightarrow x \div 2 \in E$

   F. i. $0 \in F \bigwedge 1 \in F$
      ii. $\forall x, y \quad x, y \in F \rightarrow x + y \in F$
      iii. $\forall x \quad x \in F \rightarrow \quad x \div 2 \in F$

   G. Suppose the set of real numbers G satisfies i) $3 \in G$, ii) $8 \in G$, and iii) $\forall a, b \in G, a - b \in G$. Another way to describe G is to say that 3 and 8 are members and G is *closed* under subtraction. Prove that G contains the set of all integers.

   H. i. $1/2 \in H$
      ii. If $x \in H$, then $x/2 \in H$
      iii. If $x \in H$, then $\frac{1}{1+x} \in H$

April 3, 2001 2:47 P.M.