Homework 3B   Math 1165

Profs. S. Molchanov and H. Reiter

1. Use the Principle of Mathematical Induction to prove that
$$1 + 5 + 5^2 + 5^3 + \cdots + 5^n = \frac{5^{n+1} - 1}{4} \text{ for all } n \geq 0.$$

2. Use the Principle of Mathematical Induction to prove that
$$\sum_{k=1}^{n} (2k + 3) = n(n + 4) \text{ for all } n \geq 1.$$

3. Find a formula for
$$\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right) \cdots \left(1 - \frac{1}{n^2}\right)$$
for $n \geq 2$, and use the Principle of Mathematical Induction to prove that your formula is correct.

4. Use the Principle of Mathematical Induction to prove that $2|(n^2 - n)$ for all $n \geq 0$.

5. Use the Principle of Mathematical Induction to prove that $n^2 - 5n + 3 > 0$ for all $n \geq 5$.

6. The sequence of Fibonacci numbers $f_0, f_1, f_2, \ldots$ is defined by the rule $f_0 = 0$, $f_1 = 1$, and $f_n = f_{n-1} + f_{n-2}$ for $n \geq 2$. Prove that $f_0 + f_2 + f_4 + f_6 + \cdots + f_{2n} = f_{2n+1} - 1$ for all $n \geq 0$.

7. Given a set C of real numbers that satisfies
   (a) $5 \in C \ \wedge \ 8 \in C$ (Both 5 and 8 belong to C.)
   (b) $\forall x \forall y, \quad (x \in C \quad \wedge \quad y \in C) \to (x - y \in C)$. This means that for any elements x and y of C, the difference $x - y$ also belongs to C.

   i Prove that $11 \in C$.
   ii Prove that C contains all the positive integers.

8. (The Postage Stamp problem) Suppose a and b are positive integers whose *gcd* is 1. Then there is a positive integer k, called the *conductor* of a and b, such that for any integer $n \geq k$, it is possible to express n as a nonnegative integer combination of x and y. In other words, the equation $ax + by = n$ can be solved for nonnegative integers a and b. This is the postage stamp problem with stamps of value a and b. Find the conductor K of the pair $4, 7$ and prove using the Principle of Mathematical Induction that if $n \geq K$, then n is expressible as $n = 4x + 7y$ for some nonnegative integers x and y.

9. Prove that $f_i f_{i+3} - f_{i+1} f_{i+2} = (-1)^{i+1}$ for all $i \geq 1$, where f_i denotes the i^{th} Fibonacci number.

For more induction problems, visit the website
http://www.math.uncc.edu/~hbreiter/m3166/handouts.htm **June 6, 2003**
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