Homework 3 Math 1165

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1. Prove that the number $x_n = n\,(n^2+5) \qquad n = 0, 1, 2, \ldots$ is divisible by 6.
2. Let
$$a_n = \frac{1}{n\,(n+1)}, \qquad n = 1, 2, \ldots$$
and $\qquad S_n = a_1 + a_2 + \cdots + a_n. \qquad$ Prove that
$$S_n = \frac{n}{n+1}$$
Find $\lim_{n\to\infty} S_n$.
3. Let $a_n = 3n^2 - 3n + 1$. Prove that $S_n = a_1 + a_2 + \cdots + a_n = n^3$.
4. Prove that for some integer n_0, $3^n > 5n$, for all $n \geq n_0$. Find the minimal n_0.
5. Let F_n be the Fibonacci sequence: $F_0 = F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$, for $n = 2, 3, \ldots$. Prove that $F_0 + F_1 + \cdots + F_n = F_{n+2} - 1$ for all $n \geq 0$.
6. Let $a_n = 3a_{n-1} - 2a_{n-2}$, $a_1 = 1$, $a_2 = 3$. Guess the formula for a_n and prove the result by mathematical induction.
7. Let $a_1 = \sqrt{2}$, $a_2 = \sqrt{2+\sqrt{2}}$, $\cdots$
$$a_n = \underbrace{\overbrace{\sqrt{2+\sqrt{2+\cdots\sqrt{2}}}}^{n \text{ radicals}}}$$
Find the first-order recursive relation for the sequence a_n. The rest of this problem is not part of the homework for the Fall 2000 class, but is expected for all classes thereafter. Prove each of the following about the sequence.
   (a) The sequence is *bounded above*. That is, there exists a number B such that $a_n \leq B$ for all $n \geq 1$.
   (b) The sequence is *non-decreasing*. That is, $a_n \leq a_{n+1}$ for all $n \geq 1$.
   (c) The two conditions above together imply that the sequence *converges*. That is $\lim_{n\to\infty} a_n$ exists. Find the limit.

For more induction problems, visit the website
http://www.math.uncc.edu/~hbreiter/m3166/handouts.htm