

## Homework 2 Math 1165

1. Find the least common multiple and the greatest common divisor of each of the following pairs of integers.

(a)  $m = 2001$  and  $n = 1001$ .

(b)  $m = 2^3 \cdot 3^2 \cdot 5$  and  $n = 3^2 \cdot 7^2$ .

(c)  $m = p^3 \cdot q^2 \cdot r$  and  $n = p^2 \cdot q \cdot r^3$ , where  $p$ ,  $q$ , and  $r$  are different primes.

2. Notice that we can find the number of positive integer divisors of an integer  $N$  in terms of its prime factorization. For example, the number  $2^i 3^j 5^k$  is a divisor of  $2^l 3^m 5^n$  if and only if  $i \leq l, j \leq m$  and  $k \leq n$ . Find the number of divisors of each of the integers.

(a)  $m = 2001$ .

(b)  $n = 1001$ .

(c)  $m = 2^3 \cdot 3^2 \cdot 5$ .

(d)  $n = 3^2 \cdot 7^2$ .

(e)  $m = p^3 \cdot q^2 \cdot r$ , where  $p$ ,  $q$ , and  $r$  are different primes.

3. A high school with 1000 lockers and 1000 students tries the following experiment. All lockers are initially closed. Then student number 1 opens all the lockers. Then student number 2 closes the even numbered lockers. Then student number 3 changes the status of all the lockers numbered with multiples of 3. This continues with each student changing the status of all the lockers which are numbered by multiples of his or her number. Which lockers are closed after all the 1000 students have done their jobs?

4. The French mathematician De Bauvelles (16th century) claimed that for any  $n > 1$  at least one of the numbers  $6n \pm 1$  is a prime. Say,

$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$
5 7	11 13	17 19	23 25	29 31

Show that he was wrong.

5. For what values of the unknown digit  $a$  is  $n = 47, a32$  a multiple of 3.
6. Now that you know divisibility tests for both 2 and 3, devise a divisibility test for 6.

7. Devise a divisibility test for the number 8. Recall that for divisibility by 2 we could look at just the units digit, and for 4 we need only consider the units and tens digits. How many digits must we examine for divisibility by 16? ... for  $2^k$ ?
8. Using the Euclidean Algorithm, evaluate  $g = GCD(4144, 7596)$  and  $h = LCM(4144, 7596)$ . Then solve the equation  $g = 4144x + 7596y$ , for integers  $x$  and  $y$ .
9. Write  $n = 1492$  in the base

$$b = 2, \quad b = 4, \quad b = 8.$$

What is the relationship among these representations?

10. Find the unknown digit  $x$  from the equation  $2x3_4 = 1x10_3$ .
11. Convert the numbers

$$n = (3141)_9, \quad m = (3141)_7$$

to the base  $b = 5$  and evaluate  $n + m$ ,  $n \times m$ . Use the tables from Lecture 2.

12. Construct the tables of addition and multiplication for the base  $b = 6$  and evaluate

$$(215)_6 + (304)_6, \quad (203)_6 \times (405)_6.$$

13. Let  $n = (a_i a_{i-1} a_{i-2} \dots a_0)_2$  be the binary representation of  $n$  (where each  $a_i$  is 0 or 1). Under what conditions on the  $a_i$  is  $n$  divisible by 2? by 4? by 8?
14. The Cookie Monster at each meal eats the same number of cookies from each cookie jar. Suppose there are five jars with 3, 5, 6, 18, and 22 cookies. What is the fewest number of meals required for the Cookie Monster to devour all the cookies?
15. Prove that  $n = (\overline{a_k a_{k-1} \dots a_1 a_0})_{10}$  is divisible by 11 if and only if the alternating sum  $a_0 - a_1 + a_2 - \dots \pm a_k$  is divisible by 11. Hint: look at the remainder you get when you divide various powers of 10 by 11.
16. Write down in one series all numbers between 1 and 1998 and consider this expression as a new number:

$$N = 123\dots 91011\dots 99100101\dots 10001001\dots 19971998$$

- (a) How many digits does  $N$  contain?

(b) Prove that  $9|N$ .

17. (a) Construct the multiplication and addition tables for the base  $b = 12$ .  
(b) Use the tables to multiply

$$\beta 7\alpha_{12} \cdot 389_{12}.$$

(c) Interpret  $(\alpha 9\beta 8.3\beta)_{12}$ .

18. Find the base  $-5$  representation of all the numbers from 1 to 25.
19. Find the base 4 representation of  $1/9$ . Prove your answer.
20. Find the base  $-4$  representation of  $1/3$  and then show that your answer is correct.
21. You need to bake a cake for 30 minutes, but all you have for timers is two 'minute glasses', one that measures 9 minutes and the other 13 minutes. You have just put the cake into the oven, and you start one (or both) of the glasses. How can you measure exactly 30 minutes using the two glasses. An  $n$  minute glass contains sand which slips through a small hole in the center of the glass so that after  $n$  minutes the sand has all drained through the hole.
22. Find the best (winning) move in the game of Bouton's Nim  $(14, 13, 12, 9)$ .