

Quiz 10

Find a function f satisfying the following two conditions:

- $f'(x) = 4x^2 - 7x + 5$, and
- $f(6) = 0$.

Solution. The antiderivative of $f'(x)$ is obtained as follows:
$$\int 4x^2 - 7x + 5 \, dx = \int 4x^2 \, dx - \int 7x \, dx + \int 5 \, dx = 4 \int x^2 \, dx - 7 \int x \, dx + \int 5 \, dx = 4 \cdot \frac{x^3}{3} - 7 \cdot \frac{x^2}{2} + 5x + C.$$
 Thus C must satisfy the equation $0 = f(6) = 4\frac{6^3}{3} - 7\frac{6^2}{2} + 5 \cdot 6 + C$. Solving this for C yields $C = -192$. Thus $f(x) = \frac{4x^3}{3} - \frac{7x^2}{2} + 5x - 192$.