The Asymptote Theorem for Rational Functions
Let $p(x)$ and $q(x)$ be polynomial functions, and define $r(x)$ by $r(x) = \frac{p(x)}{q(x)}$. Let

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

and

$$q(x) = b_m x^m + b_{m-1} x^{m-1} + \cdots + b_1 x + b_0.$$

Also, suppose that $p(x)$ and $q(x)$ have no common zeros. Then

(a) Horizontal Asymptote
   i. If $m < n$, there is no horizontal asymptote.
   ii. If $m = n$, then $y = \frac{a_n}{b_m}$ is the horizontal asymptote.
   iii. If $m > n$, then $y = 0$ is the horizontal asymptote.

(b) Vertical Asymptotes
   i. Every zero of the denominator $q(x)$ determines a vertical asymptote. If $r_1, r_2, \ldots, r_k$ are zeros of $q(x)$, then the lines $x = r_1$, $x = r_2, \ldots,$ $x = r_k$ are all vertical asymptotes of $r(x)$.