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The Center and Cyclicity Problems
A Computational Algebra Approach

In the last three decades, advances in methods for investigating polynomial ideals and their varieties have provided new possibilities for approaching two long-standing problems in the theory of differential equations: the Poincaré center problem and the cyclicity problem (the problem of bifurcation of limit cycles from singular trajectories).

Using a computational algebra approach, this work addresses the center and cyclicity problems as behaviors of dynamical systems and families of polynomial systems. The text first lays the groundwork for computational algebra and gives the main properties of ideals in polynomial rings and their affine varieties; this is followed by a discussion regarding the theory of normal forms and stability of differential equations. The center and cyclicity problems are then explored in detail.

The book contains numerous examples, pseudocode displays of all the computational algorithms, historical notes, nearly two hundred exercises, and an extensive bibliography. Completely self-contained, it is thus suitable mainly as a textbook for a graduate course in the subject but also as a reference for researchers.

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$$\begin{aligned} \tilde{P}(x, y) &= i \left(x - \sum_{(p,q) \in S} a_{pq} x^p y^q \right) \\ \tilde{Q}(x, y) &= -i \left(y - \sum_{(p,q) \in S} b_{pq} x^p y^q \right) \end{aligned}$$

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$$g_k = -i \left[\sum_{\substack{s_1 + s_2 = 0 \\ s_1, s_2 \geq -1}}^{2k-1} [(s_1 + 1) a_{k-s_1, k-s_2}] \right]$$