Math 1241 - Spring 2008

Second Practice Midterm, March 2008
Instructor: Catalin Turc

You must show **all** your work to receive credit.
Important: No books, graphing calculators, or notes are allowed. Cheating may result in failure of course (don't even think about it). Please read each question carefully, show all your work, and check afterwards that you have answered all questions correctly. Any crossed work will be disregarded (even if correct). Write **one** clear answer with a coherent derivation. Good luck!

[**1**] Given the function $f(x)$, find a formula for $f'(x)$ using the definition with limits:

$$a) f(x) = \frac{1-x}{2+x}$$

$$a) f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1-x-h}{2+x+h} - \frac{1-x}{2+x}}{h}$$

$$= \lim_{h\to 0} \frac{(1-x-h)(2+x)-(1-x)(2+x+h)}{h(2+x)(2+x+h)}$$

$$= \lim_{h\to 0} \frac{(2-2h-x-xh-x^2)-(2-x+h-xh-x^2)}{h(2+x)(2+x+h)}$$

$$= \lim_{h\to 0} \frac{-3h}{h(2+x)(2+x+h)} = -\frac{3}{(2+x)^2}.$$

$$b) f(x) = \sqrt{x}.$$

$$b) f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}$$

$$= \lim_{h\to 0} \frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{h(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{h\to 0} \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0} \frac{h}{h(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}.$$

[**2**] The equation of motion of a particle is $s = t^3 - 3t$, where s is in meters and t is in seconds. Find
(a) the velocity and acceleration as functions of t;
(b) the acceleration after $2s$
(c) the acceleration when the velocity is 0
(d) when is the particle moving forward?

(a) $v(t) = s'(t) = 3t^2 - 3$ and $a(t) = v'(t) = 6t$
(b) after $2s$ the acceleration is $a(2) = 12m/s^2$
(c) the velocity being equal to 0 means that we have to solve the equation $v(t) = 0$ that is $3t^2 - 3 = 0 \iff (t-1)(t+1) = 0 \Rightarrow t = 1$ since that is the physical value for the time. At time $t = 1$ the acceleration is $a(1) = 6$
(d) The particle is moving forward when the velocity is positive, which is $3t^2 - 3 > 0$, and we get $t > 1$.

[**3**] Compute the derivatives of the following functions using the appropriate rules of differentiation:

$$a)y = \frac{e^x}{x^2}$$

a) Use the quotient rule and get $y' = \frac{e^x(x-2)}{x^3}$

$$b)y = \frac{3x-1}{2x+1}$$

b) Use the quotient rule again and get $y' = \frac{5}{(2x+1)^2}$

$$c)y = (x + e^x)(3 - \sqrt{x})$$

c) Use the product rule and get $y' = (1 + e^x)(3 - \sqrt{x}) - \frac{x+e^x}{2\sqrt{x}}$

$$d)y = \frac{1 + \sin x}{x + \cos x}$$

d) Use the quotient rule and get $y' = \frac{c \cos x}{(x = \cos x)^2}$.

[**4**] Compute the derivatives of the following functions using appropriate rules of differentiation and the chain rule:

$$a)y = xe^{-x^2}$$

a) Use the product rule to get $y' = e^{-x^2} + x(e^{-x^2})'$. For this derivative we apply the chain rule $(e^{-x^2})' = e^{-x^2} \cdot (-x^2)' = -2xe^{-x^2}$. Finally we obtain $y' = e^{-x^2}(1 - 2x^2)$.

$$b) y = \sqrt{\frac{x-1}{x+1}}.$$

b) First write $y = \left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$. Then apply the chain rule to get

$$y' = \frac{1}{2}\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}-1} \cdot \left(\frac{x-1}{x+1}\right)'$$

$$= \frac{1}{2}\left(\frac{x-1}{x+1}\right)^{-\frac{1}{2}} \frac{2}{(x+1)^2}$$

$$= \left(\frac{x-1}{x+1}\right)^{-\frac{1}{2}} \frac{1}{(x+1)^2} = \frac{1}{\sqrt{(x-1)(x+1)^3}}.$$