Math 2164 Spring 2008

Extra Credit Problems, Feb 2008
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[**1**] (25 points) An algorithm for solving a system is usually measured in flops (or floating point operations). A *flop* is one arithmetic operation (+, -, *, /) on two real floating point numbers. Show that for a $n \times (n+1)$ augmented matrix, the reduction to echelon form takes $2n^3/3 + n^2/2 - 7n/6$ flops. Show that a further reduction to reduced row echelon form needs at most n^2 flops.
(*Hint:* use the fact that $\sum_{k=1}^{N} k^2 = 1^2+2^2+3^2+\cdots+N^2 = N(N+1)(2N+1)/6$ and $\sum_{k=1}^{N} k = 1+2+3+\cdots+N = N(N+1)/2$.)

[**2**] (25 points) Let A be a $n \times n$ matrix that has the property that the entry from the i^{th} row and j^{th} column equals to $\min(i, j)$. For instance, when $n = 5$ the matrix looks like

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 & 3 \\ 1 & 2 & 3 & 4 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}.$$

Compute the reduced row echelon form of such matrices for all values of n.

[**3**] (50 points) Let T be the linear transformation that maps $\mathbf{R}^3$ into $\mathbf{R}^3$ given by $T(\vec{x}) = A\vec{x}$ where

$$A = \begin{pmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{pmatrix}.$$

a) Show that there exists a line that goes through the origin that T maps onto itself (*Hint:* Solve the matrix equation $A\vec{x} = 9\vec{x}$);

b) Show that there exists a plane that goes through the origin that T maps onto itself (*Hint:* Solve the matrix equation $A\vec{x} = 2\vec{x}$);

c) If T is a linear transformation of $\mathbf{R}^2$ into $\mathbf{R}^2$, is the following assertion true: There exists a line through the origin that T maps onto itself?